BLACK HOLE EVAPORATION: ENTROPY ANALYSIS AND GUP CORRECTIONS

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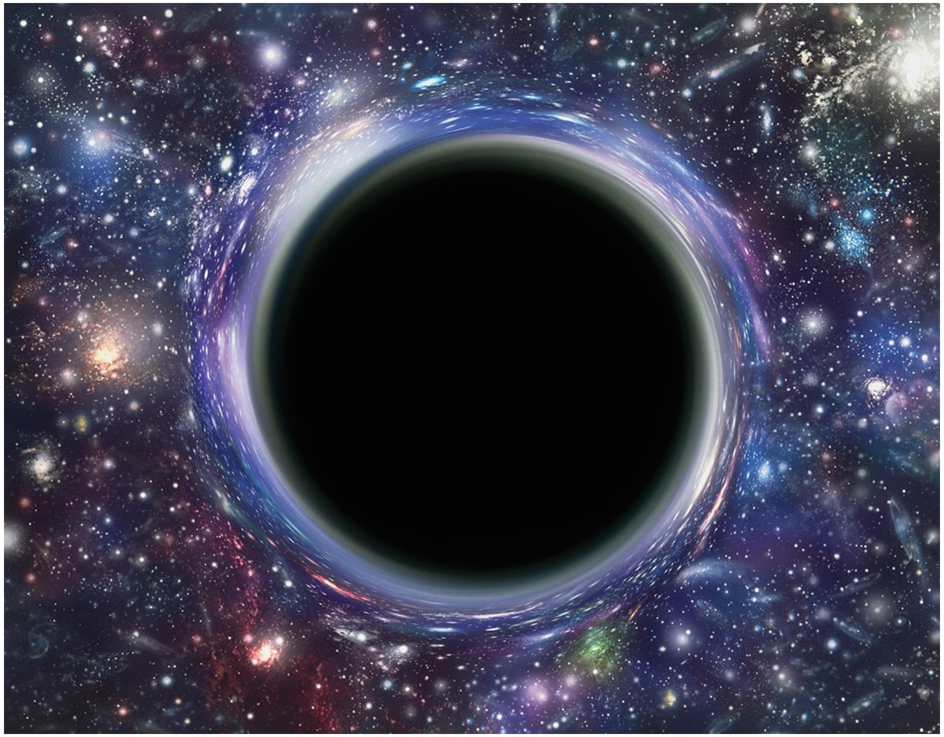
OUTLINE

- On burning a lump of coal: standard entropy budget in thermodynamic
- Coarse-graining the entropy
- Extension to black holes: Entropy/Information flux in Hawking radiation
  - Thermodynamic entropy
  - Entanglement entropy
    - Bipartite system
    - Tripartite system
    - Multipartite system
- Quantum corrections to the Hawking flux
- Discussion

BLACK HOLES

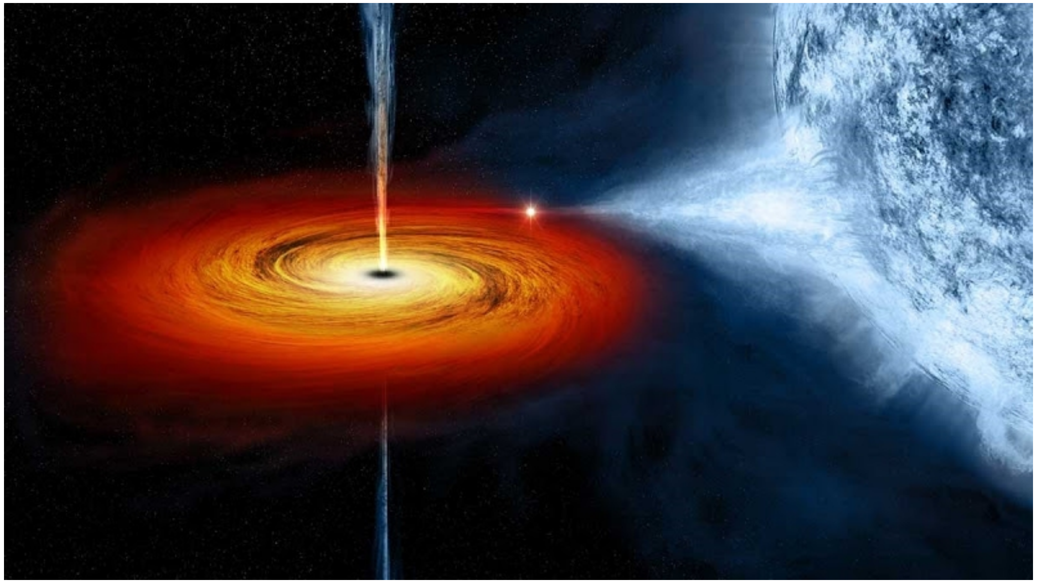


- Black holes from gravitational collapse
- Solutions of Relativity
  - Regions of spacetime with extremely high curvature
  - Vacuum solution
- Event horizon: surface that isolate black hole and from where gravity is so strong that neither light can escape
- Breakdown of General Relativity → singularity! (infinite density)
- Tidal forces: Chaotic spacetime (BKL conjecture)

- Black holes acreate all the surrounding matter

- No-hair theorem:
  completely characterized by M, Q, J

- They also evaporate due to quantum effects (Hawking radiation)



$$T = \frac{\hbar \kappa}{2\pi k_B c} \propto \frac{1}{M} \quad \Rightarrow \quad S = \frac{k_B c^3}{4\pi G \hbar} A$$

Bekenstein entropy

- 4th laws of black hole thermodynamics

$M \rightarrow E$ ✓ $\kappa \rightarrow T$ ✓ $A \rightarrow S$ ✓

- Entropy as a lack of information → and Bekenstein entropy is huge!

Quantum information theory

THE BLACK HOLE INFORMATION PUZZLE

- Complete evaporation due to Hawking radiation
- Hawing flux is thermal radiation $\longrightarrow$ it does not carry almost information
- Quantum mechanically: initially is a pure state $\longrightarrow$ finally is a mixed state

$\Downarrow$

Non-unitary process! (not allowed in QM): Do not preserve entropy

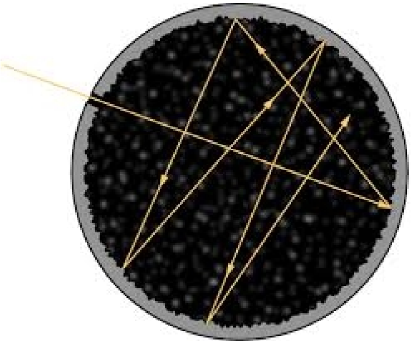
$\Downarrow$

Entropy as a lack of information: Where is the information lost?

- This problems continue completely unsolved: Very different proposals
- Theory of Quantum Gravity?

WHAT AM I GOING TO ADDRESS IN THIS TALK?

- Understand the entropy issues regarding black hole formation (coarse-graining)
- Understand the entropy/information flow
- Simple well-understood process: Burning a lump of coal



Each photon carries (approx) 3 bits of information

- Schwarzschild black hole emitted quanta emitted the same!
- We got a lot of insight from quantum gravity corrected system
  - Modifications of the information flow at last stages of evaporation

- Quantum information in black hole evaporation

- The quantum concept of entropy: Entanglement entropy

  - These analisys in the past gave rise to paradoxes → firewalls

  - We realized something was missed in the picture: Environment

- We have introduced a new model to deal with this issue (new tools needed to be developed)

  - We have obtained a continuous flux of entropy/information along the whole evaporation, without paradoxes

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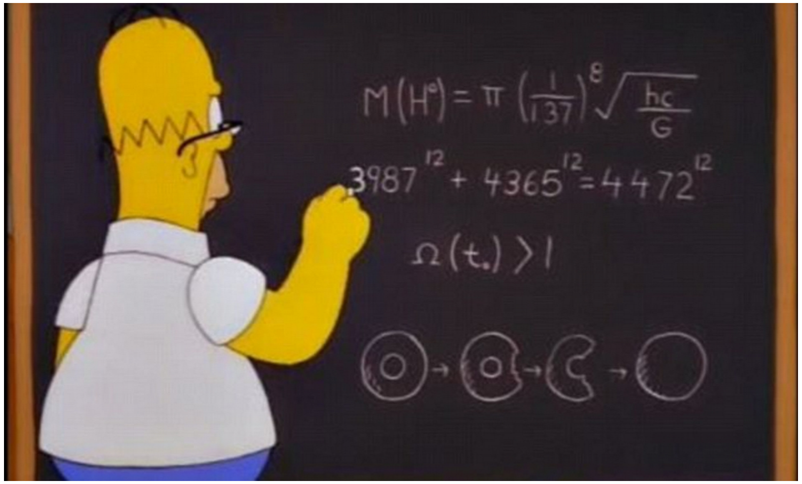
$\Rightarrow$ Let's go!!



ON BURNING A LUMP OF COAL

- Standard statistical mechanics applied to a furnace with a small hole $\longrightarrow$ blackbody radiation (Planck spectrum implies some coarse graining)
  - Transfer of thermodynamic entropy to the radiation field: $S = \frac{E}{T} = \frac{\hbar\omega}{T}$

- Consider the effect of coarse graining the (von Neumann) entropy

$$S_{coarse\ grained} = S_{before\ coarse\ graining} + S_{correlations} \longrightarrow I_{correlations}$$

- The average energy per photon: $\langle E \rangle = \hbar \langle \omega \rangle = \hbar \frac{\int \omega f(\omega) d\omega}{\int f(\omega) d\omega} = \frac{\pi^4}{30\zeta(3)} k_B T$

- Consequently, we can calculate the average entropy per photon (and the standard deviation) [A. A-S, Matt Visser, PLB 757 (2016) 383)]

$$\boxed{\hat{S}_2 = 3.9 \pm 2.5} \text{ bits/photon}$$

- Since we know the underlying physics is unitary $\longrightarrow$ this entropy is compensated by an equal "hidden information"

COARSE GRAINING THE ENTROPY

- Explicit and calculable processes that takes an arbitrary system and monotonically and controllably drives the entropy to the maximum value

- It is relevant when we deal with physical issues of information

  - Price of coarse graining: some properties are hidden in the system

  - Measure of uncertainty $\longrightarrow$ entropy as a lack of information

Coarse graining the classical Shannon entropy

- Continuum $\longrightarrow$ continuum: Diffusion process
- Continuum $\longrightarrow$ discretium: Box averaging
- Discretium $\longrightarrow$ discretium: Aggregation/averaging

Coarse graining the quantum Von Neumann entropy

- Maximal mixing

- Tunable Partial traces

- Full/tunable partial decoherence

EXTENSION TO BLACK HOLES

★ unitarity ★ apparent/trapping horizons ★ complete evaporation

THERMODYNAMIC ENTROPY IN THE HAWKING FLUX

- Loss of Bekenstein entropy of a Schwarzschild black hole

$$\frac{dS}{dN}=\frac{dS/dt}{dN/dt}=\frac{d\left(4\pi k_B GM^2/\hbar c\right)/dt}{dN/dt}=\frac{k_B\pi^4}{30\zeta(3)}$$

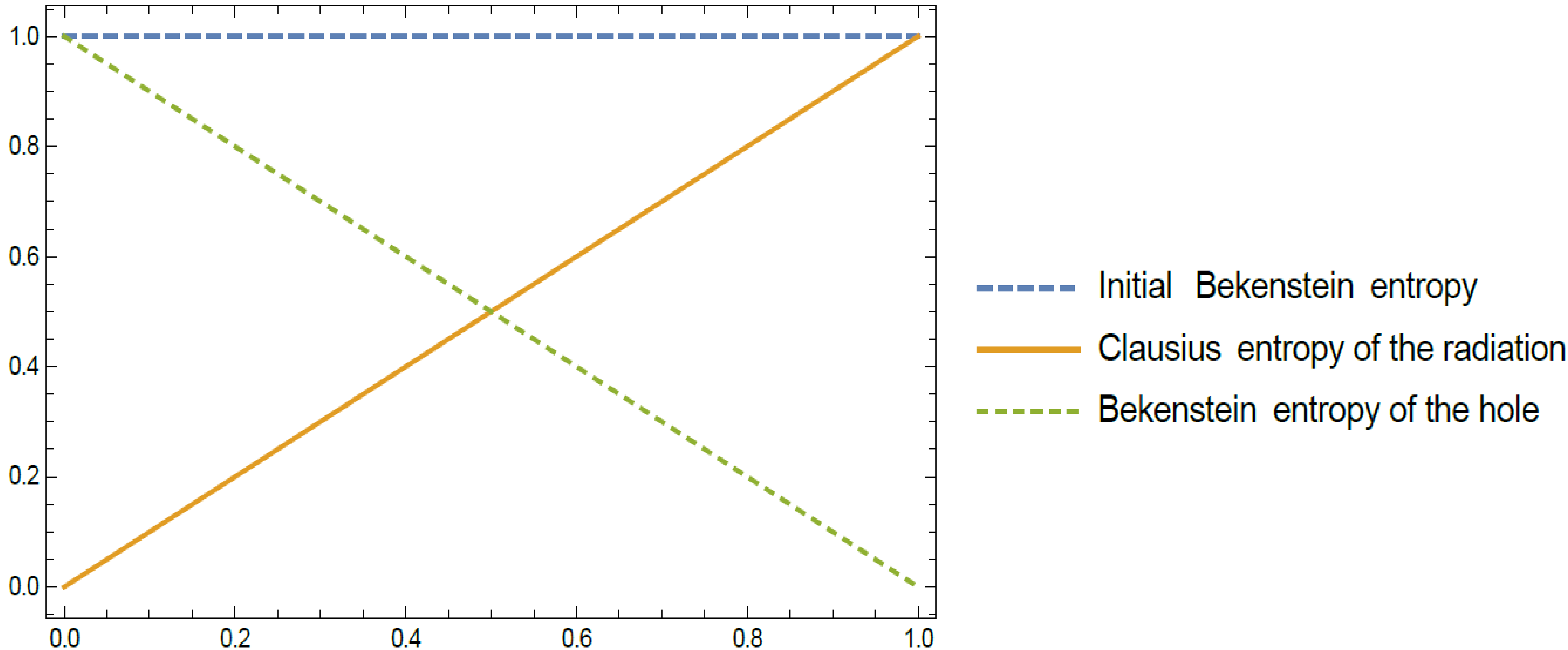
- Thermodynamic entropy gain (Clausius entropy gain) of the external radiation field per emitted quanta $\longrightarrow$ entropy gain of the Hawking flux

$$\frac{dS}{dN}=\frac{dE/dT_H}{dN}=\frac{\hbar\langle\omega\rangle dN}{T_H dN}=\frac{k_B\pi^4}{30\zeta(3)} \quad \Rightarrow \quad \frac{d\hat{S}_2}{dN}\approx 3.896976 \quad \text{bits/quanta}$$

- The Hawking radiation is essentially (adiabatically) transferring Bekenstein entropy from the hole into Clausius entropy of the radiation field
  [A. A-S, M. Visser, PLB 776 (2018) 10-16]

- Estimation of the total number of emitted massless quanta $\frac{dN}{dM}=\frac{30\zeta(3)}{\pi^4}\frac{8\pi GM}{\hbar c}$

- Throughout the evaporation process we have $S_{Bekenstein}(t)+S_{Clausius}(t)=S_{Bekenstein,0}$



- So, semiclassically everything holds together very well

- Sparsity of the Hawking flux: Average time between emission of successive Hawking quanta is many times larger than the natural timescale set by the energies of the emitted quanta [F. Gray et al., CQG 33 (2016) 115003]

ENTANGLEMENT ENTROPY IN THE HAWKING FLUX

- Average subsystem entropies [D. N. Page, PRL 71(1993) 1291]

- Consider a Hilbert space that factorizes $H_{AB} = H_A \otimes H_B$

  Pure state $\rho_{AB} = |\psi\rangle\langle\psi|$ $\longrightarrow$ Subsystem density matrices $\rho_A = tr_B(|\psi\rangle\langle\psi|)$

  Subsystem von Neumann entanglement entropy $\hat{S}_A = -tr(\rho_A \ln \rho_A)$

- Uniform average over all pure states, taking:

  $n_1 = dim(H_A)$, $n_2 = dim(H_B)$ and $m = min[n_1, n_2]$

  The central result $\hat{S}_{n_1, n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \leqslant \ln m$

- Average subsystem entropy is very close to its maximum possible value

  Strict bound (combined with our results): $\hat{S}_{n_1, n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \in \left(\ln m - \frac{1}{2}, \ln m\right)$

Bipartite entanglement: black hole + Hawking radiation

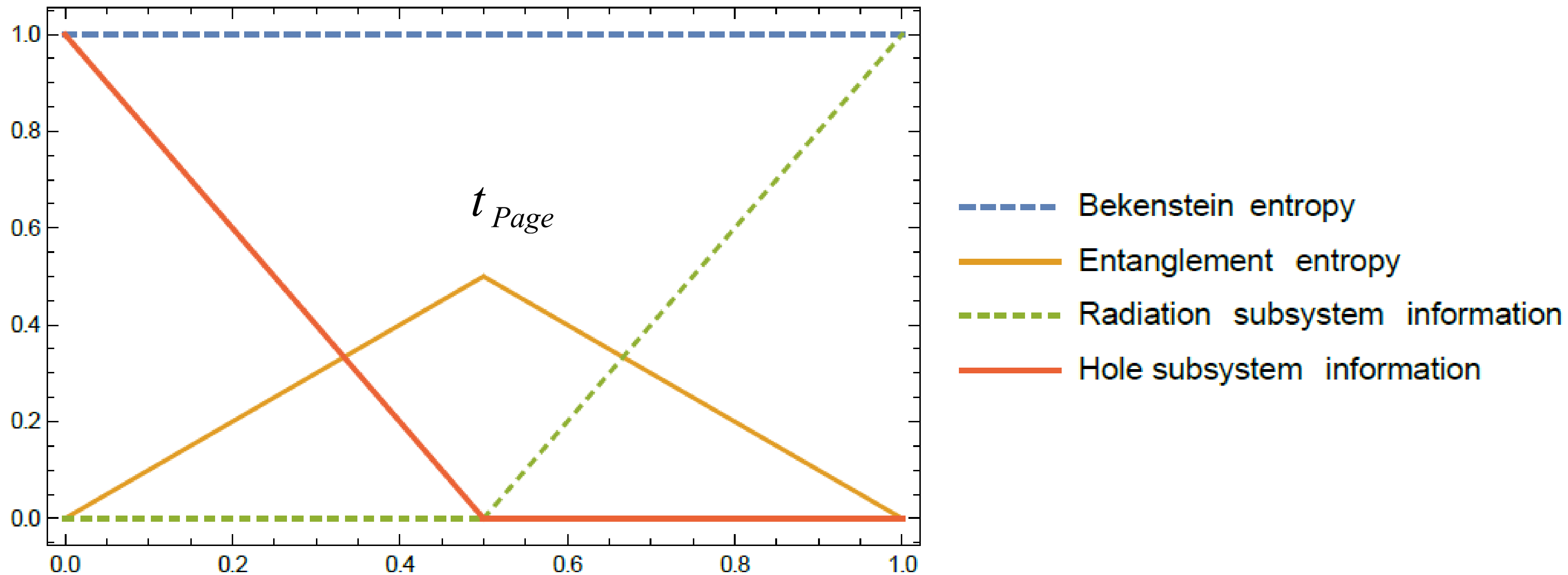
[D. N. Page, PRL 71(1993) 3743]

- "Closed box" argument
- Initially there is no yet any Hawking radiation $\begin{cases} H_R & \text{trivial} \\ H_H & \text{enormous} \end{cases} \longrightarrow (\hat{S}_{n_H, n_R})_0 = 0$
- After the black hole has completely evaporated: H_H is trivial $\longrightarrow (\hat{S}_{n_H, n_R})_\infty = 0$
- At intermediate times both dimensionalities are nontrivial $\longrightarrow (\hat{S}_{n_H, n_R})_t \neq 0$
  - Since the evolution is assumed unitary: total Hilbert space is constant

$$n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty} \longrightarrow (\hat{S}_{n_H, n_R})_t = \ln \left\{ n_H(t), \frac{n_{H_0}}{n_H(t)} \right\}$$

Maximized when $n_H(t) \approx \sqrt{n_{H_0}} \longrightarrow \hat{S}_{n_H, n_R}(t = t_{Page}) \approx \frac{1}{2} \ln n_{H_0}$

➢ Page curve:



- Pages defines a novel asymmetric version of the subsystem information (no direct physical interpretation) ⟶ Mutual information and other measures of entanglement (such as negativity, tangle or concurrence)

- It is the shape of this curve that underlies much of the modern discussion surrounding the "information puzzle"

- Subsystem entropy is initially zero ⟶ tension with Bekenstein entropy

- If we entangle the black hole with the environment, then the total state is not pure

- What happens with these results?

  - The late radiation is maximally entangled with the early radiation and the hole subsystems

⬇

Monogamy of entanglement

⬇

Firewalls

- We though that the point is the considered “closed box” system

  - May be it is more appropriate to consider a tripartite system, including the interaction with the environment

Tripartite entanglement: bh + Hawking radiation + rest of the universe

[A. A-S, Matt Visser, PLB 757 (2016) 383)]

- The Hilbert space is now $H_{HRE} = H_H \otimes H_R \otimes H_E$

- Take the entire universe be in a pure state $S_{HRE}(t) = 0$

  And now the subsystem entropies $S_H(t) = S_{ER}(t)$, $S_R(t) = S_{HE}(t)$, $S_E(t) = S_{HR}(t)$

- Initially $S_{H_0} = S_{E_0}$, $S_{R_0} = 0 = S_{HE_0}$

- Once the black hole has completely evaporated $S_{H_\infty} = 0 = S_{ER_\infty}$, $S_{R_\infty} = S_{E_\infty}$

- The evolution is assumed unitary, with the unitary time evolution operator factorized as

$$U_{HRE} = U_{HR}(t) \otimes U_E(t)$$

Therefore $\begin{cases} n_{E_0} = n_{E_\infty} \equiv n_E \\ n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty} \end{cases}$

- We make an additional assumption: That the Bekenstein entropy can be identified with the average entanglement entropy

- Then, the entropies:

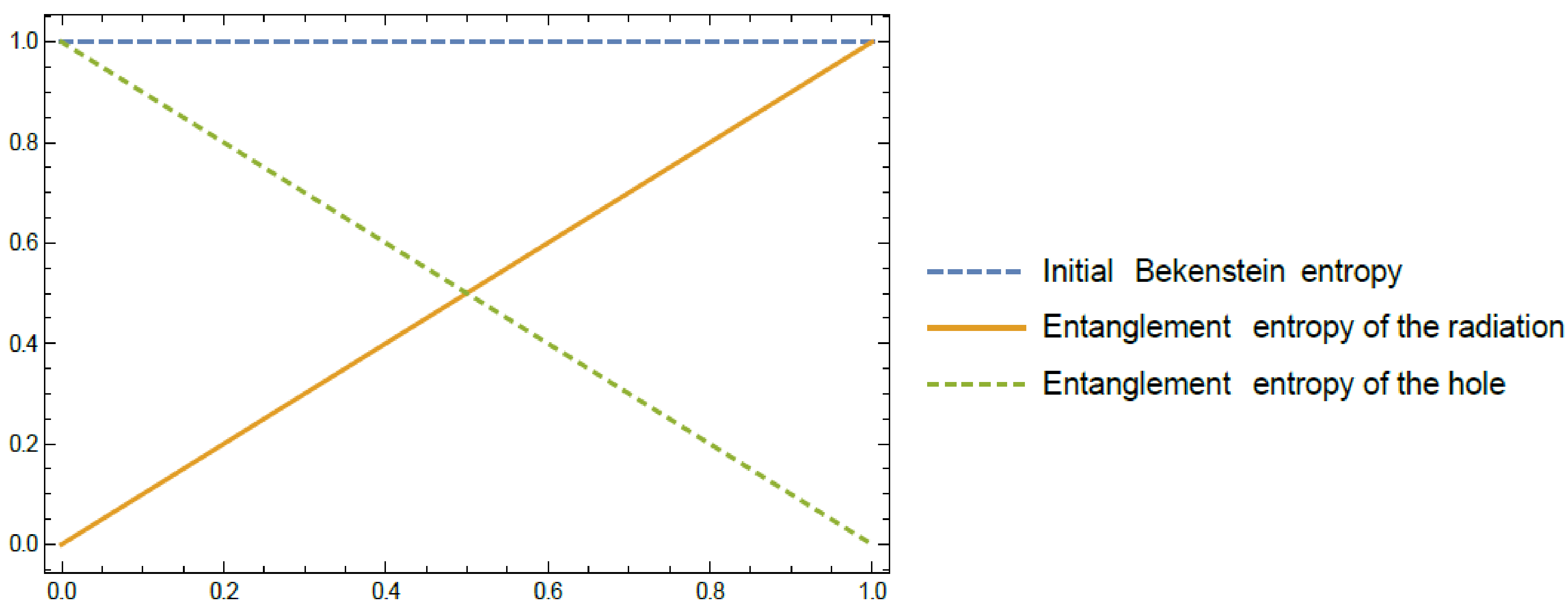
  - $\langle \hat{S}_H(t) \rangle \approx \ln min[n_H(t), n_R(t) n_E] \approx \ln n_H(t)$
  - $\langle \hat{S}_R(t) \rangle \approx \ln min[n_R(t), n_H(t) n_E] \approx \ln n_R(t)$

$$\langle \hat{S}_H(t) \rangle + \langle \hat{S}_R(t) \rangle \approx \ln[n_H(t) n_R(t)] = \ln n_{H_0}$$

$$\hat{S}_{Bekenstein}(t) + \langle \hat{S}_{Hawking\ radiation}(t) \rangle \approx \hat{S}_{Bekenstein,0}$$

- The "rest of the universe environment": the extent to which the subsystem is entangled

$$\langle \hat{S}_E(t) \rangle \approx \ln min[n_E(t), n_H(t) n_R] \approx \ln n_{H_0} \approx \hat{S}_{Bekenstein,0}$$



Mutual information

- For the tripartite system: $I_{H:R} = S_H + S_R - S_{HR}$ $\longrightarrow S_E$

- Averaging over the pure states in the total system we obtain

$$\left\langle \hat{I}_{H:R} \right\rangle \leqslant \frac{n_{H_0}}{2\mathrm{n}_E} \leqslant \frac{1}{2}$$

$\Longrightarrow$ So the average mutual information never exceeds ½ nat throughout the entire evaporation process

Multi-partite entanglement

- The Hilbert space is now $H = \otimes_{i=1}^{N} H_i$
- Then, the partial traces are defined as

$$\rho_{ij} = tr_{H / H_i \otimes H_j} \rho$$

- The mutual information is given by

$$\left\langle \hat{I}_{(ijk...):(pqr...)} \right\rangle \leqslant \frac{n_{ijk...}^2 \, n_{pqr...}^2}{2\mathrm{n}} \leqslant \frac{1}{2}$$

➡ So the average mutual information between any two "small" collections of subsystems in the multi-partite pure-state system never exceeds ½ nat as long as the rest of subsystem collection is dominant
[A. A-S, Matt Visser, PRA 96 (2017) 052302]

GUP IMPACT ONTO THE ENTROPY BUDGET

- Modification of the Hawking flux when we take into account quantum gravity effects (when the size approaches the Planck length)
  [A. A-S, M. Dabrowski and H. Gohar, PRD 97 (2018) 044029]

- It is possible to directly apply the Generalized uncertainty principle (GUP)

$$\Delta x \Delta p = h[1 + \alpha^2 (\Delta p)^2] \quad \text{where} \quad \alpha = \alpha_0 \frac{l_p}{\hbar}$$

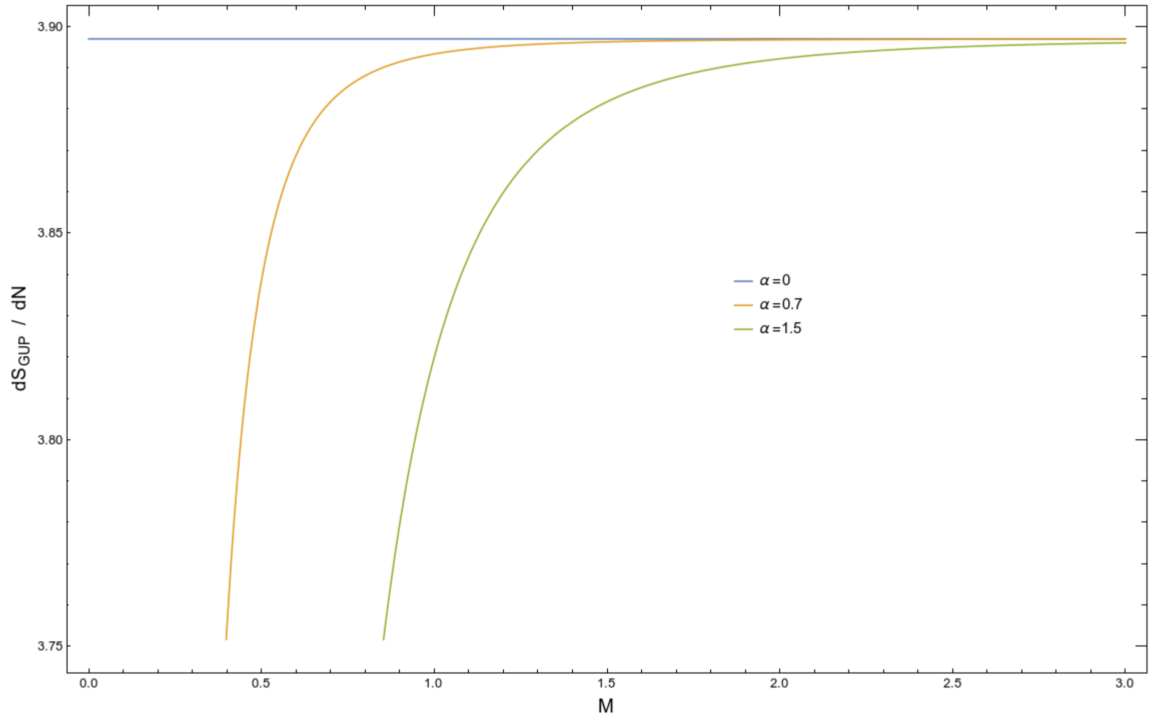
- We obtain a modified Hawking temperature

$$T_{GUP} = T\left[1 + \left(\frac{2\alpha\pi k_B}{c} T\right)^2 + O(\alpha^4)\right]$$

$$\Downarrow$$

$$S_{GUP} = S - \frac{\alpha^2 c^2 m_p^2 k_B \pi}{4} \ln\left(\frac{S}{S_0}\right) + O(\alpha^4)$$

- This leads to a modification of the entropy budget $\frac{dS_{GUP}}{dN} \simeq \frac{k_B \pi^4}{30\zeta(3)} \left[1 - \left(\frac{\alpha c}{4} \right)^4 \left(\frac{m_p^2}{M} \right)^4 \right]$



The entropy budget per emitted particle decreases when the black hole approaches the Planck size

- This leads to a modification of total number of emitted quanta

$$N_{GUP} \simeq \frac{30\zeta(3)}{\pi^4} \left[\frac{4\pi}{m_p^2} M^2 - \frac{\alpha^2 c^2 m_p^2 \pi}{4} \ln\left(\frac{M}{M_0} \right)^2 \right]$$

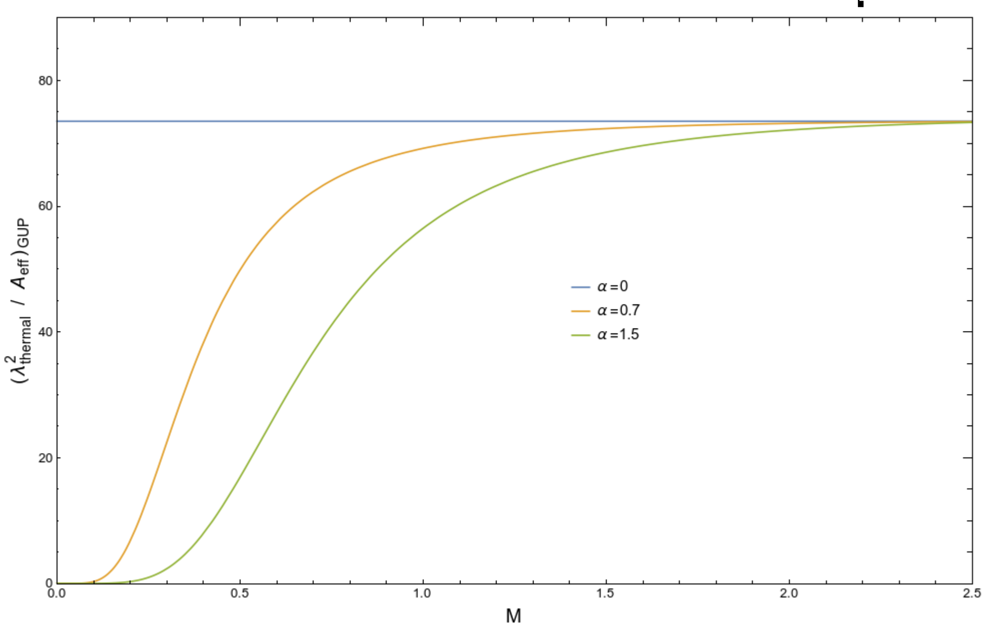
SPARSITY OF HAWKING RADIATION

- Several dimensionless quantities that gave the ratio between an average time between the emission of two consecutive quanta and the natural time scale [F. Gray et al., CQG 33 (2016) 115003]

$$\eta = C\frac{\lambda_{thermal}^2}{gA} \gg 1 \qquad \text{where} \quad \lambda_{thermal} = 2\pi\hbar c/(k_B T)$$

- GUP modification

$$\frac{\lambda_{thermal}^2}{A_{eff}}\Big|_{GUP} = \frac{64\pi^3}{27}\frac{M^6}{\left[M^2 - \left(\frac{\alpha c}{4}\right)^2 m_p^4 \ln\left(\frac{M^2}{M_0^2}\right)\right]\left[M^2 + \left(\frac{\alpha c}{4}\right)^2 m_p^4\right]^2}$$



Hawking flux is no longer sparse

DISCUSSION

- There is no "information puzzle" in burning a lump of coal $\longrightarrow$ entropy budget = "hidden information" in the correlations
- We have modelled the coarse-graining procedure in a quantifiable and controllable manner $\longrightarrow$ starting point
- In a black hole system, we calculated the classical thermodynamics entropy and the Bekenstein entropy and they compensate perfectly
- Quantum mechanically, the bipartite Page system give rise to not well-undestood physics $\longrightarrow$ new tripartite model, which quantum entropy completely agree with the classical expected results (and multi-partite extension)
- When we restrict attention to a particular subsystem we perceive an amount of entanglement entropy (a loss of information)= entropy that is codified in the correlations between the subsystems $\longrightarrow$ no weird physical effects
- We have investigated quantum gravity modifications to the entropy and temperature of an evaporating black hole expressed by the GUP

Thank you for your attention!

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[8] A. A-S, Matt Visser, "*Gravitational collapse: The big coarse graining*", work in progress.