Constraining Cosmological Models through Gravitational Waves Observations

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CANTATA
Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

BAYLOR UNIVERSITY

Goal

- We investigate and constrain cosmological scenarios that can describe the observed Universe as a whole

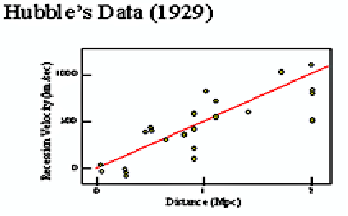
- Astrophysical cosmology has become a precision science with a huge amount of data. The advancing gravitational wave multi-messenger astronomy opens a new era

Talk Plan

- 1) Observational Cosmology: the Standard Model of Cosmology.
- 2) Standard Model of Cosmology. Do we need new physics?
- 3) We can modify the Universe content, or/and the gravitational theory.
- 4) Use of various observational data (SnIa, CMB, BAO, H(z), LSS etc) in order to constrain the proposed theories.
- 5) Torsional modified gravity: A good candidate.
- 6) GWs: basic properties and evolution.
- 7) Gravitational wave astronomy, and multi-messenger astronomy: a novel tool to test General Relativity and cosmological scenarios in great accuracy.

Observations

- Cosmological Principle "axiom" (indirect result): the Universe is homogeneous and isotropic
- Hubble (1929): The Universe expands



$$v = H\ r$$ $$H_0 \approx 70\ km\ s^{-1}\ Mpc^{-1}$$

- Alpher, Bethe, Gamow (1948): The Universe begun to expand from a very high-density and high-temperature state towards less dense and hot states. Hoyle named the theory "The Big Bang Theory".

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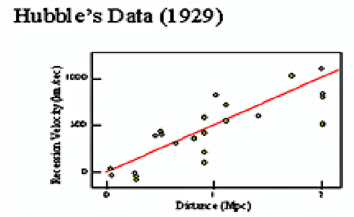

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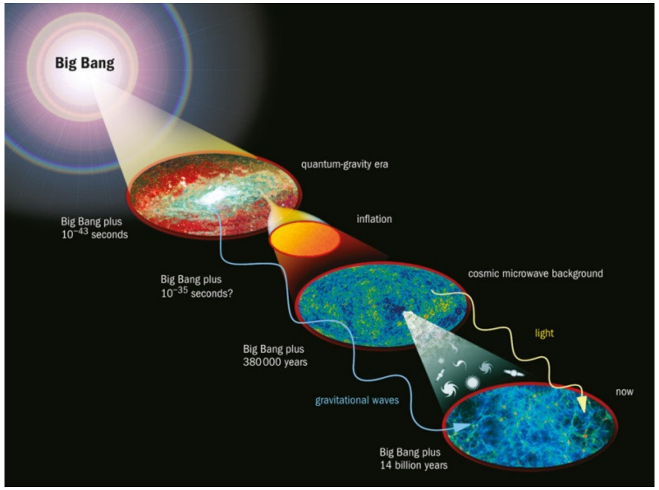
- Theoretical Problems:

- I) Horizon problem: Why points at opposite directions have the same properties
- II) Flatness problem: Why the universe is today almost spatially flat Ω_k ~0.001. It must have started with ~ 10^{-50}!
- Monopole problem: They are not observed.

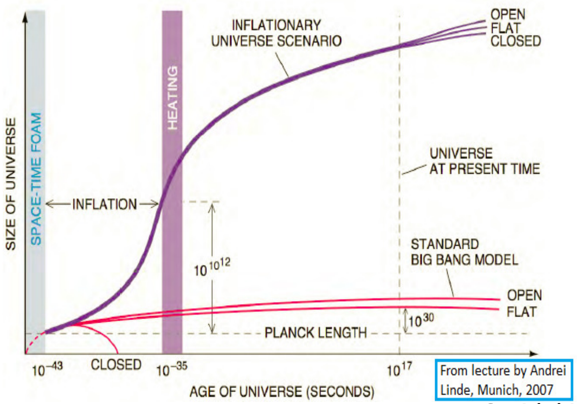
Inflation

- Kazanas, Guth, Linde (1982): The Universe $10^{-36}\,\mathrm{sec}$ after the Big Bang, through some mechanism went into an exponential expansion up to $10^{-32}\,\mathrm{sec}$ increasing in size ~ 10^{30} times: Inflation.

- I) The observable Universe is a tiny part of the total one, and originates from a small, causally connected region.
- II) Due to the huge expansion, the spatial curvature became almost zero.
- III) Due to the huge expansion the monopoles spread in all regions, and thus our own, observable universe, has at most one.

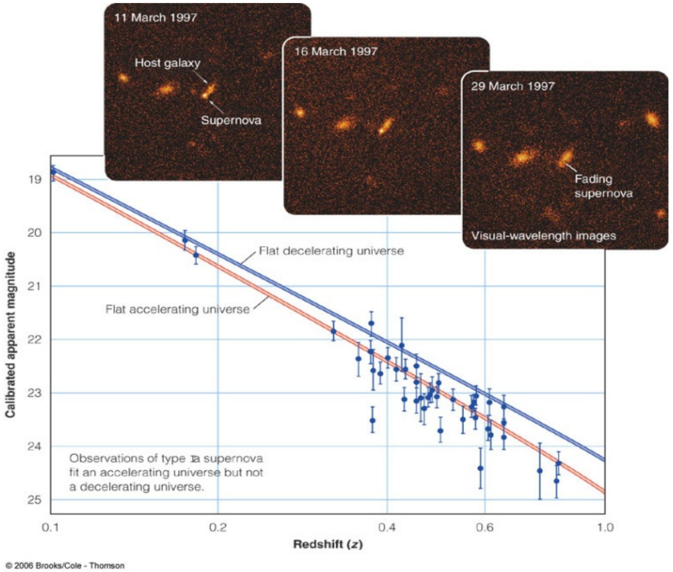


Inflationary Universe



Dark Energy

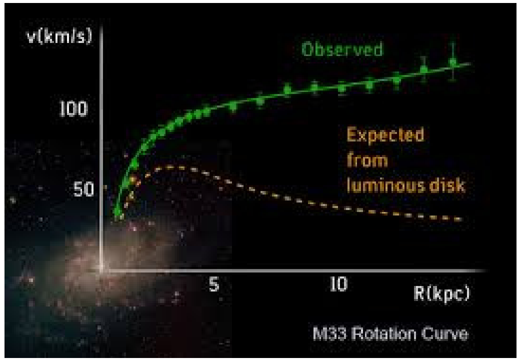
- The accelerated expansion is verified by independent observations, Supernovae type Ia (SNIa), Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO), Large Scale Structure (LSS), etc



- Around 70% of the total energy density of the Universe is this unknown dark energy (it does not interact electromagnetically).
- Possible explanation: The cosmological constant Λ (Einstein's "greatest blunder"). A term that produces the extra "repulsion".

Dark Matter

- Galaxy rotation curves:



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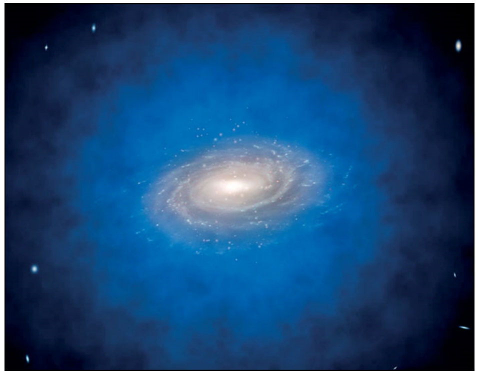
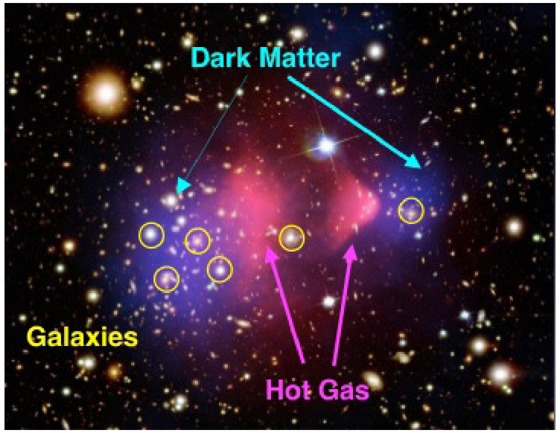


- Bullet cluster (collision of two galaxy clusters)

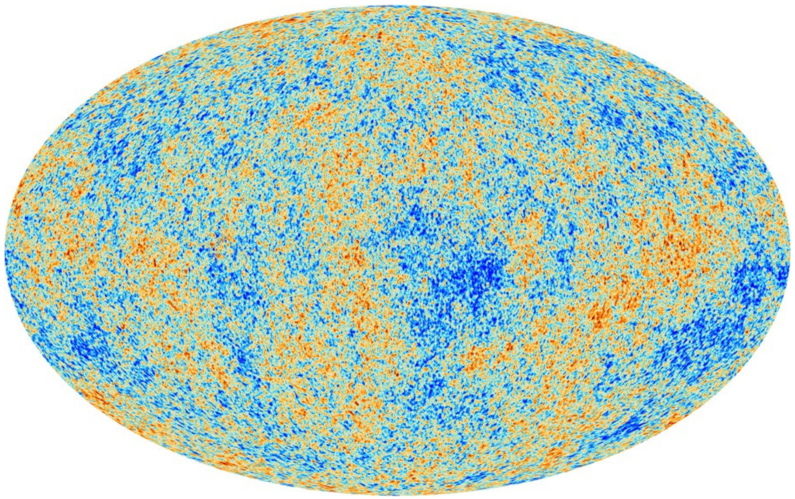


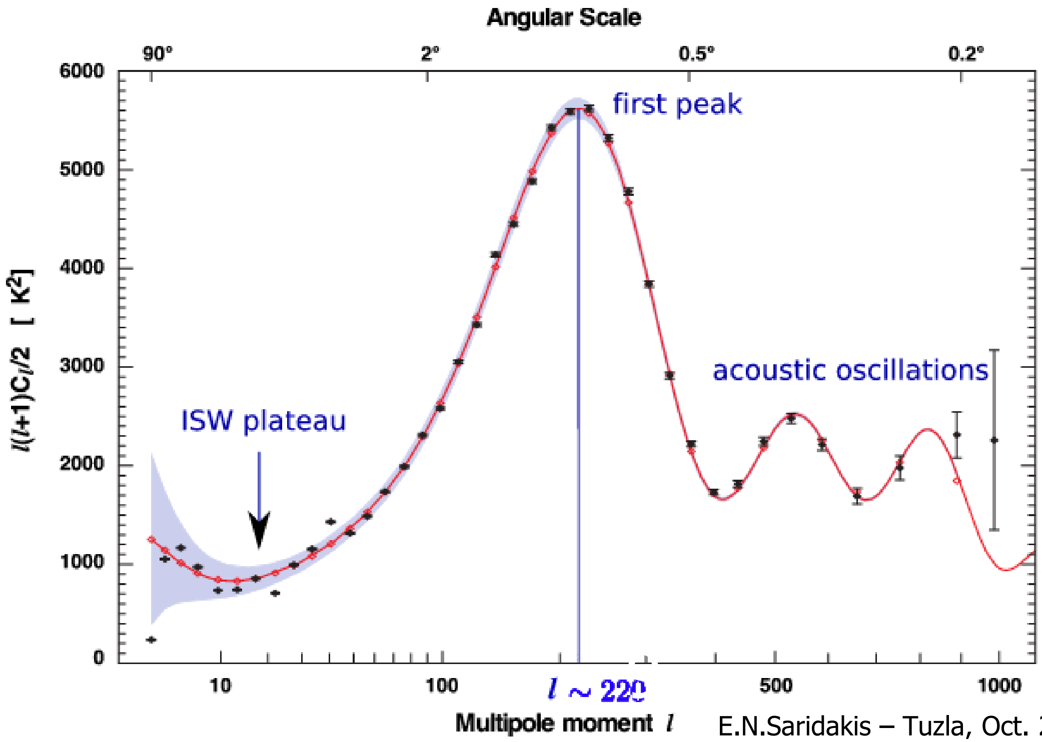
⇒

- 80% of matter is an "unknown" dark matter (it does not interact electromagnetically)!

Cosmic Microwave Background radiation

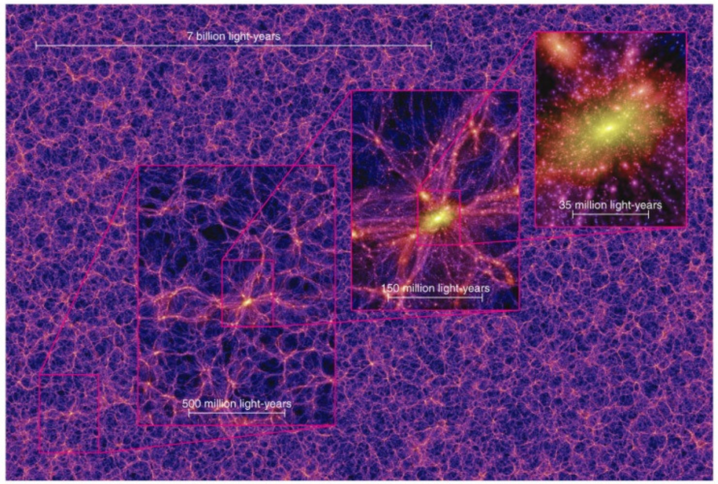
- From the fluctuation spectrum we extract information: The first peak provides the spatial curvature (it results to flat universe), the second peak the baryon energy density parameter, the third peak the dark matter energy density parameter, etc.





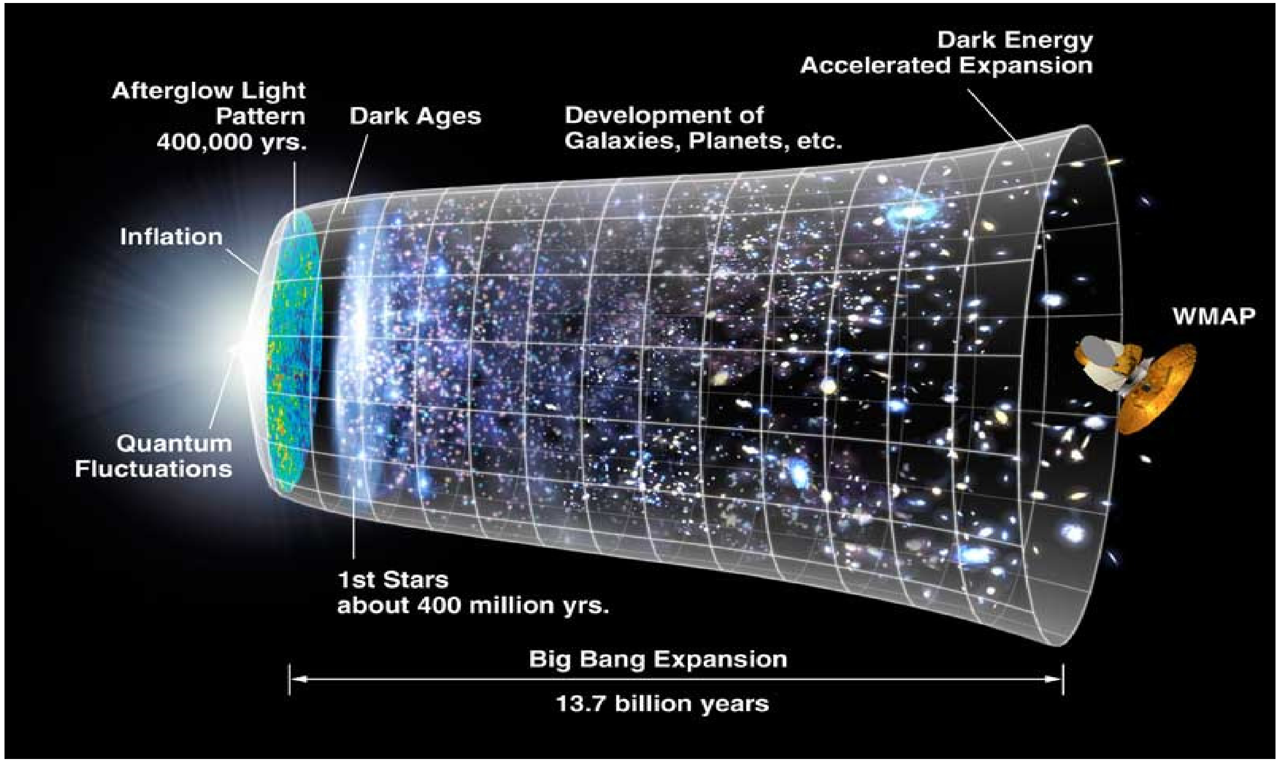
Inflation can also explain CMB and seeds of LSS

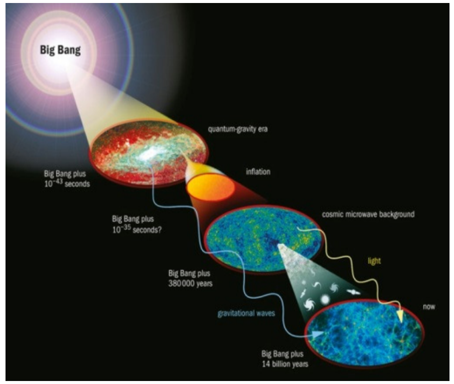
- Additional success: Inflation provides the necessary primordial fluctuations, which letter gave the Large Scale Structure of matter:

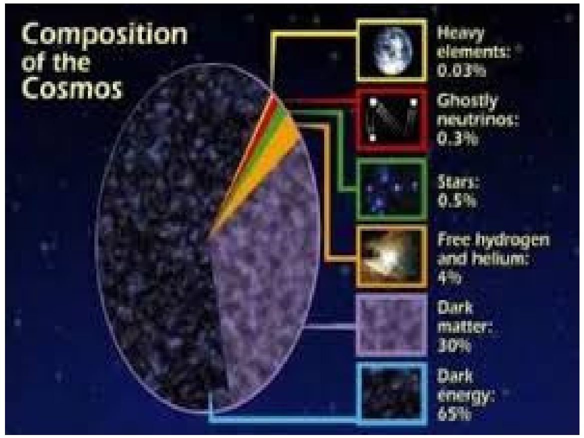


Summary of Observations

The Universe history:

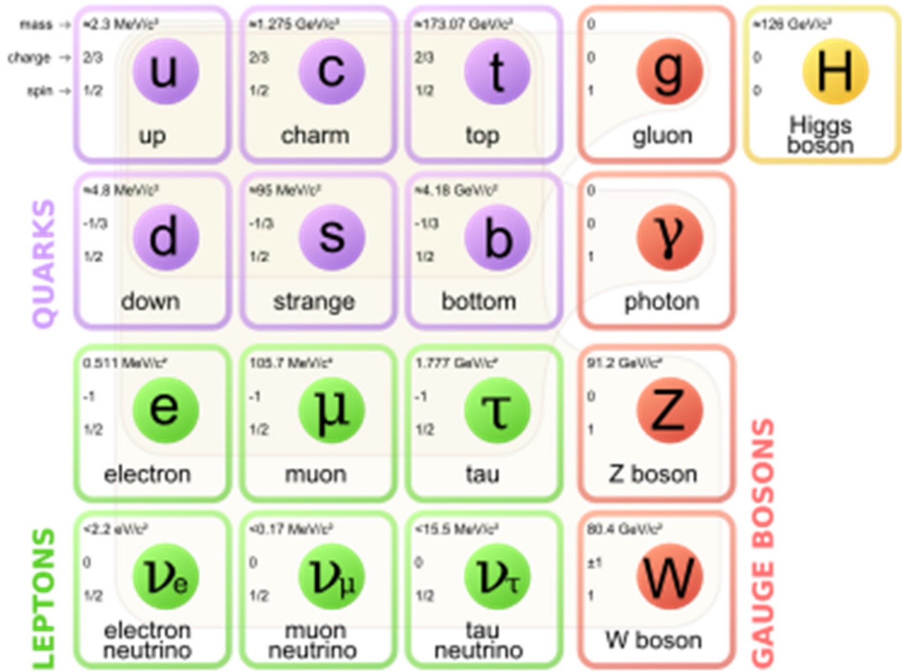






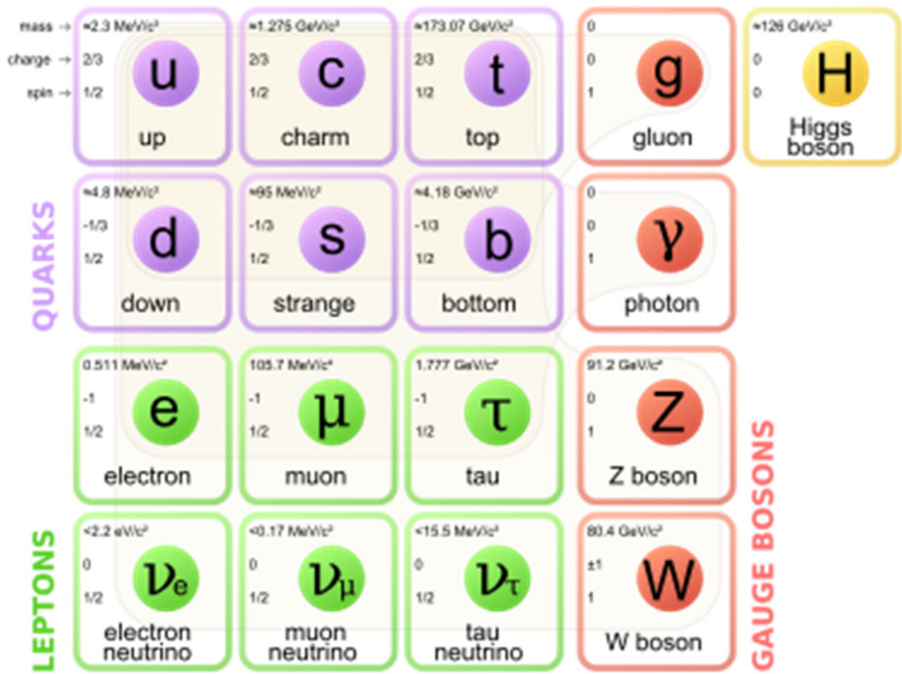
Knowledge of Physics

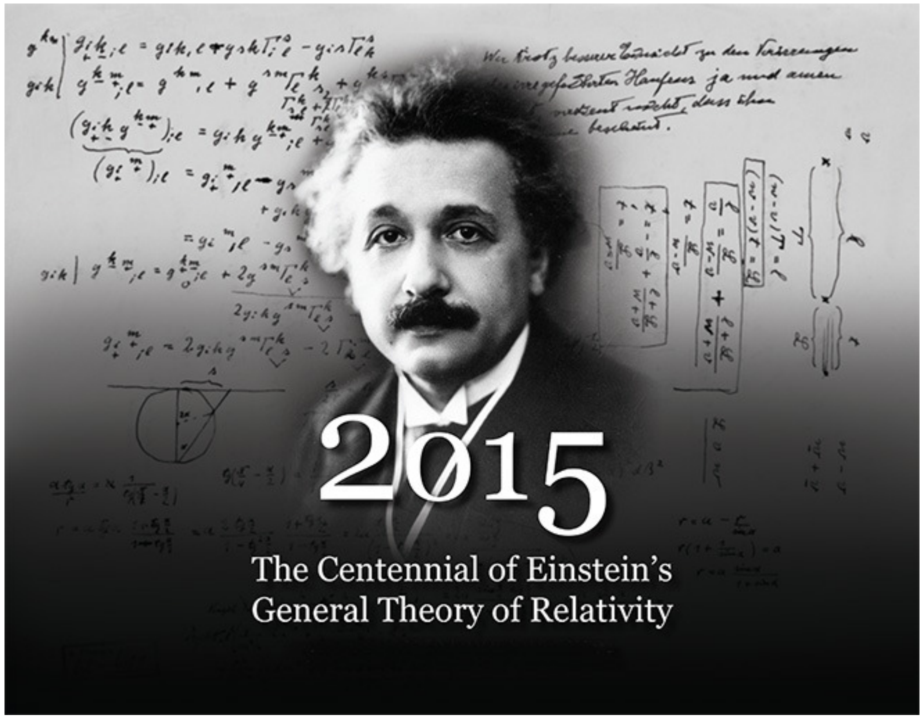
Knowledge of Physics: Standard Model



Knowledge of Physics

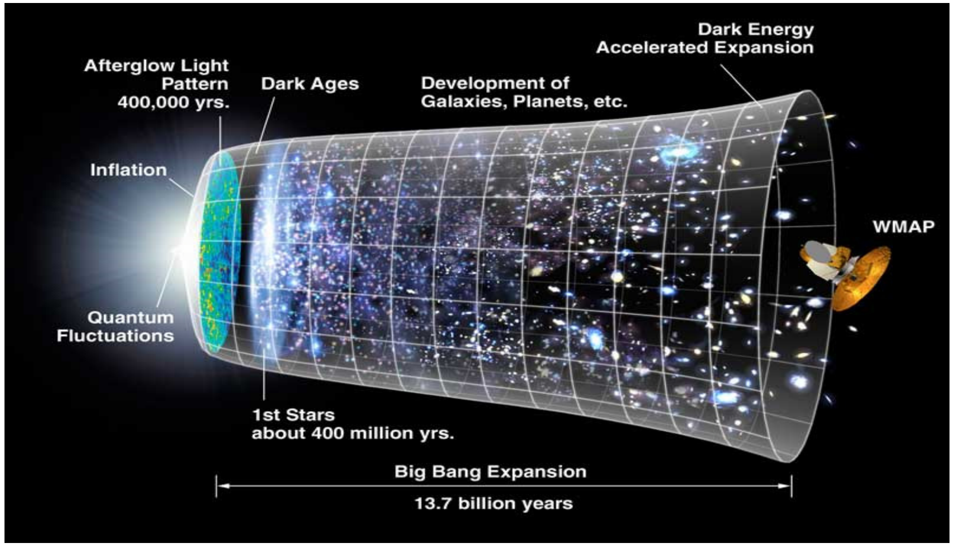
Knowledge of Physics: Standard Model + General Relativity

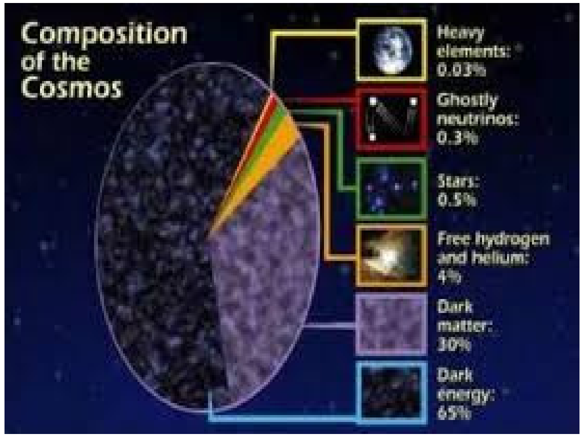




Modified/new knowledge of physics

So can our knowledge of Physics describes all these?





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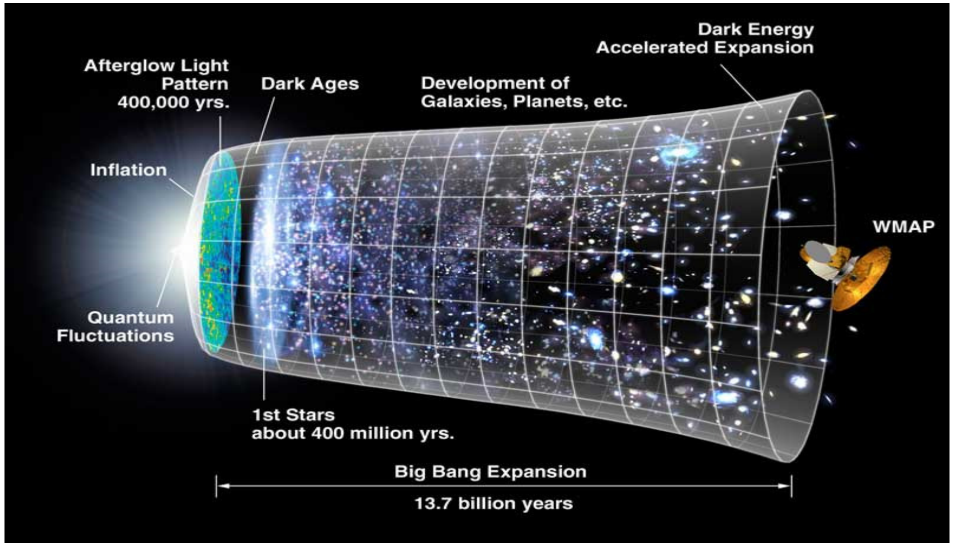

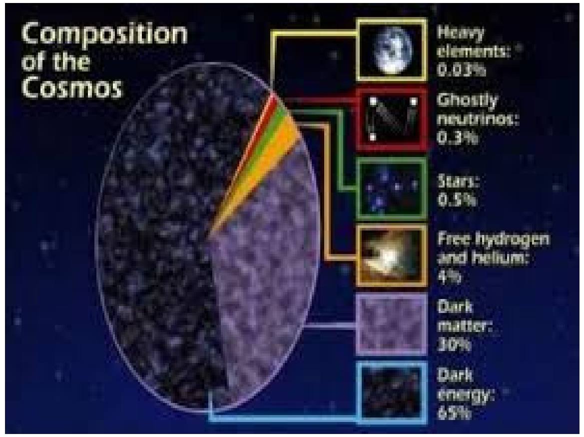

Most probably, no!

We definitely need new physics for Inflation and Dark matter. Maybe for dark energy.

Cosmology

- A successful cosmological model must:

1) Describe the evolution of the universe at the background level
2) Describe the evolution of the universe at the perturbation level

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1) Describe the evolution of the universe at the background level
2) Describe the evolution of the universe at the perturbation level

- ΛCDM paradigm seems to succeed in both, at post-inflationary eras

- Open issues:

1) The cosmological-constant problem. Calculation of Λ gives a number 120 orders of magnitude larger than observed.
   Worst error in the hist~~ory of physics, hist~~~~ory of science, h~~istory
2) How to describe primordial universe (inflation)
3) Tensions with some data sets, e.g. $H0$ and $f\sigma 8$ data

Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$

- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right)$$
$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a H(z) and Ωm(z) and wDE(z) in agreement with observations (SNIa, BAO, CMB shift parameter, H(z) etc)

Cosmology-perturbations

- Perturbation evolution: $\ddot{\delta}+2H\dot{\delta}-4\pi G_{\rm eff}\,\rho\,\delta\approx 0$ where $\delta\equiv\delta\rho/\rho$

  where $G_{\rm eff}(z,k)$ is the effective Newton's constant, given by

  $$\nabla^2\phi\approx 4\pi G_{\rm eff}\rho\,\delta,$$

  under the scalar metric perturbation $ds^2=-(1+2\phi)dt^2+a^2(1-2\psi)d\vec{x}^{\,2}$

- Hence: $\delta''+\left(\frac{(H^2)'}{2\,H^2}-\frac{1}{1+z}\right)\delta'\approx\frac{3}{2}(1+z)\frac{H_0^2}{H^2}\frac{G_{\rm eff}(z,k)}{G_N}\,\Omega_{0m}\delta$

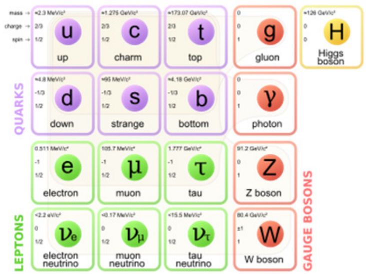
  with $f(a)=\frac{dln\delta}{dlna}$ the growth rate, with $f(a)=\Omega_{\rm m}(a)^{\gamma(a)}$ and $\Omega_{\rm m}(a)\equiv\frac{\Omega_{0m}\,a^{-3}}{H(a)^2/H_0^2}$

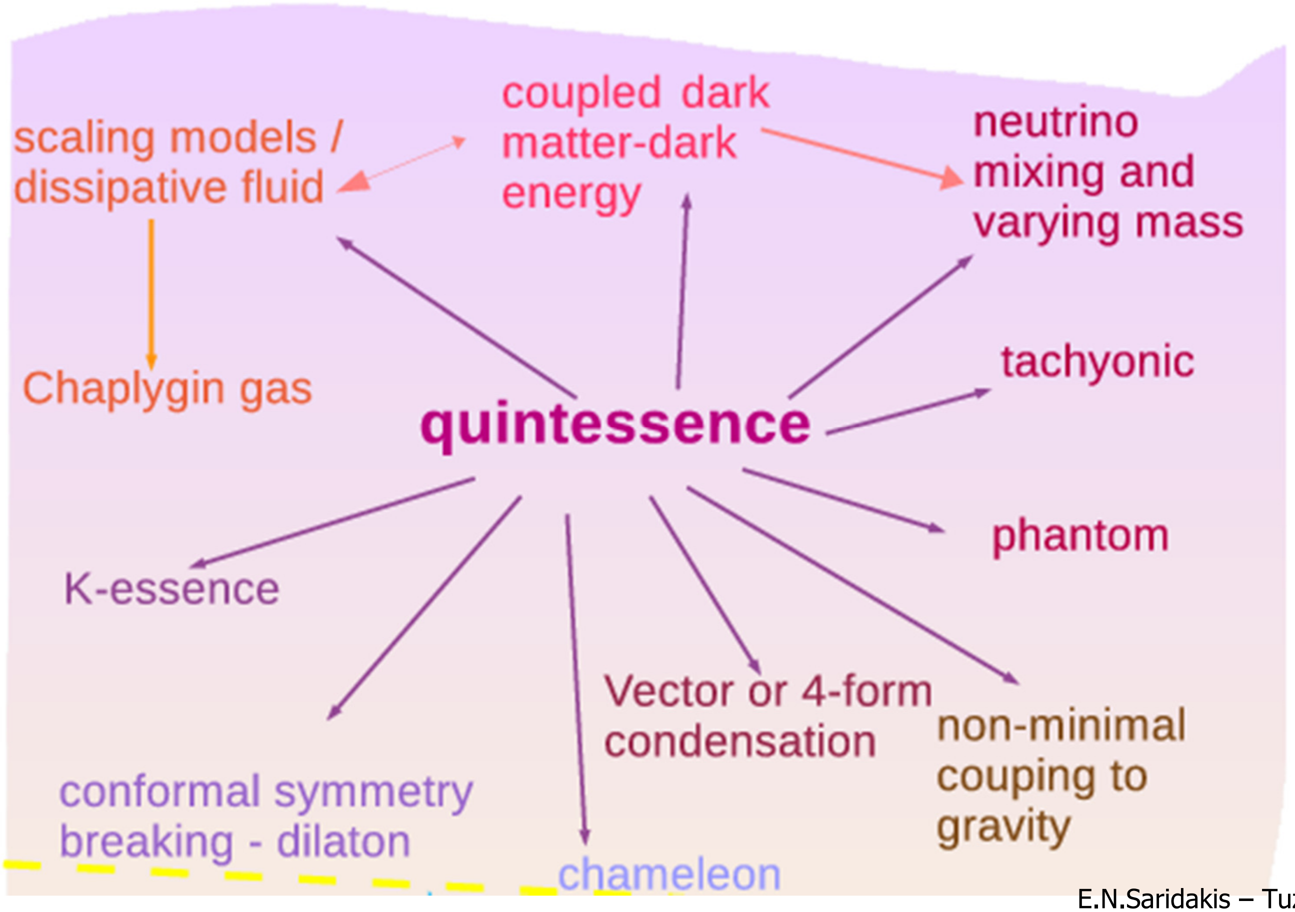
- One can define the observable: $f\sigma_8(a)\equiv f(a)\cdot\sigma(a)=\frac{\sigma_8}{\delta(1)}\,a\,\delta'(a)$

  with $\sigma(a)=\sigma_8\frac{\delta(a)}{\delta 1}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R=8h^{-1}{\rm Mpc}$, and σ8 its value today.

Dark Energy-Inflation

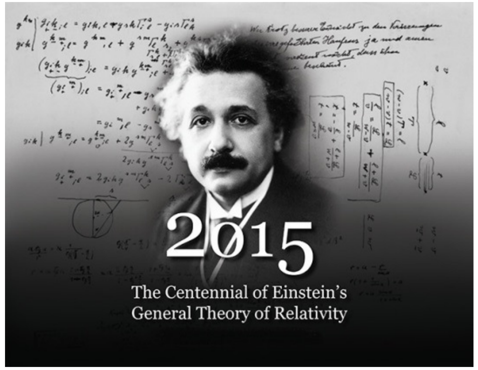
- Add a scalar field φ in the Universe content





General Relativity

- Einstein 1915: **General Relativity**:



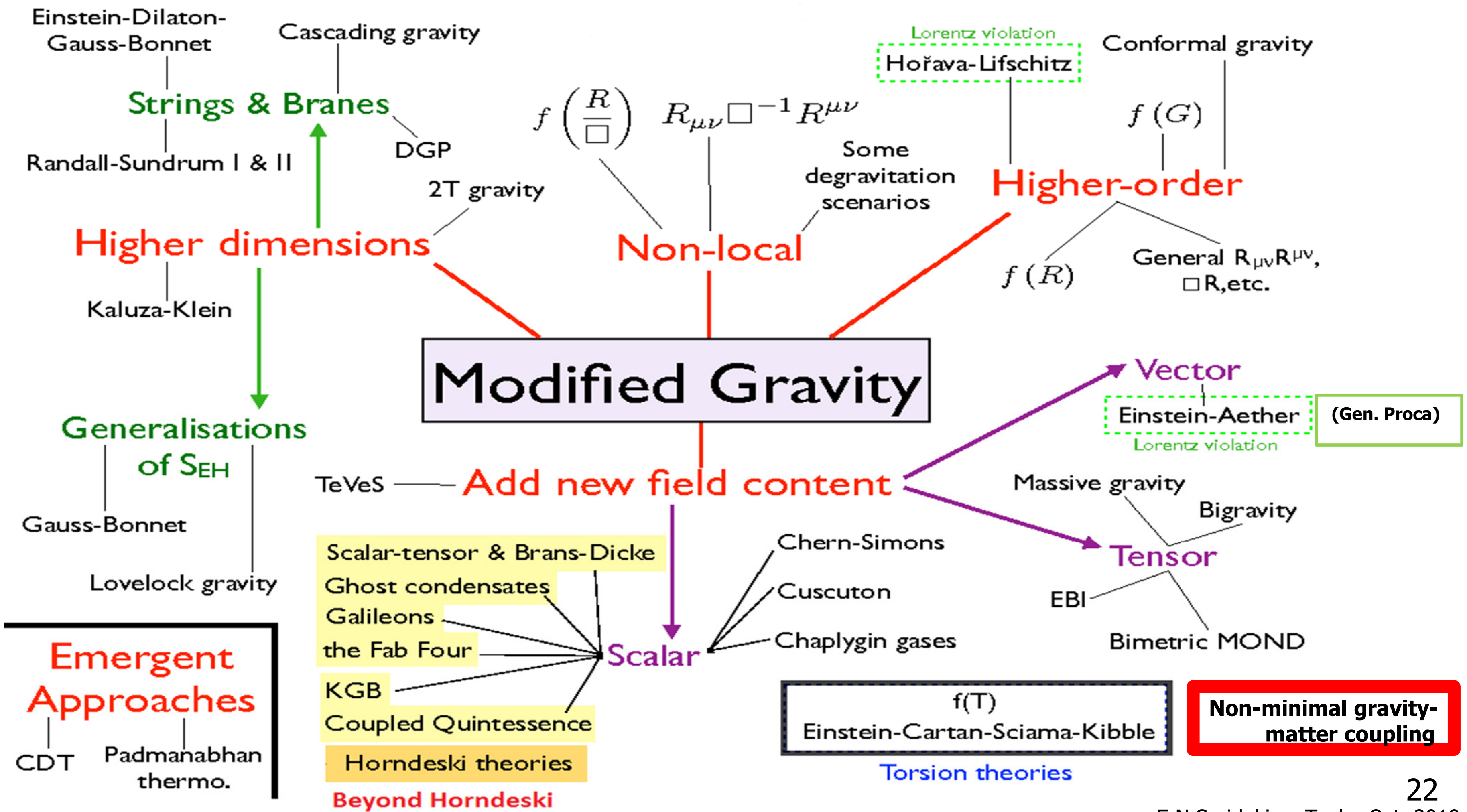
energy-momentum source of spacetime Curvature

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-2\Lambda\right]+\int d^4x\, L_m\left(g_{\mu\nu},\psi\right)$$

$$\Rightarrow \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = 8\pi G\, T_{\mu\nu}$$

with $\quad T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$

Modified Gravity



Inflation: scalar field

$$\mathrm{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \mathrm{V}(\varphi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,,$$

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0$$

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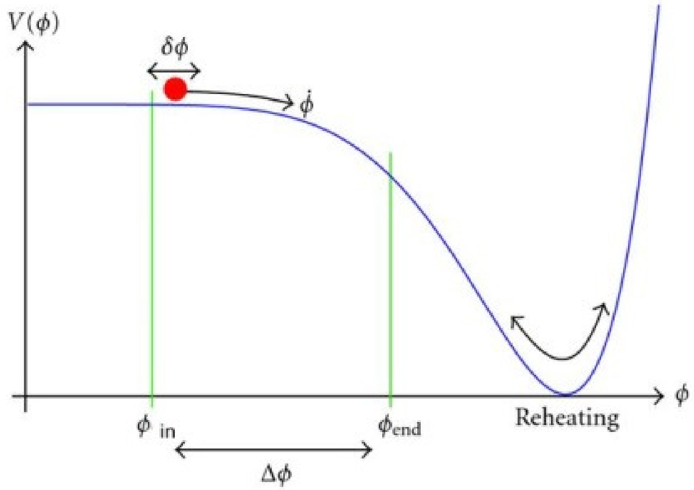
- Slow-roll conditions: $\dot{\phi}^2/2 \ll V(\phi)$ and $|\ddot{\phi}| \ll 3H|\dot{\phi}|$

$$H^2 \simeq \frac{8\pi V(\phi)}{3m_{\rm pl}^2}\,,$$

$$3H\dot{\phi} \simeq -V_\phi(\phi)$$

$$N \equiv \ln\frac{a_f}{a} = \int_t^{t_f} H\mathrm{d}t \simeq \frac{8\pi}{m_{\rm pl}^2}\int_{\phi_f}^{\phi}\frac{V}{V_\phi}\mathrm{d}\phi$$

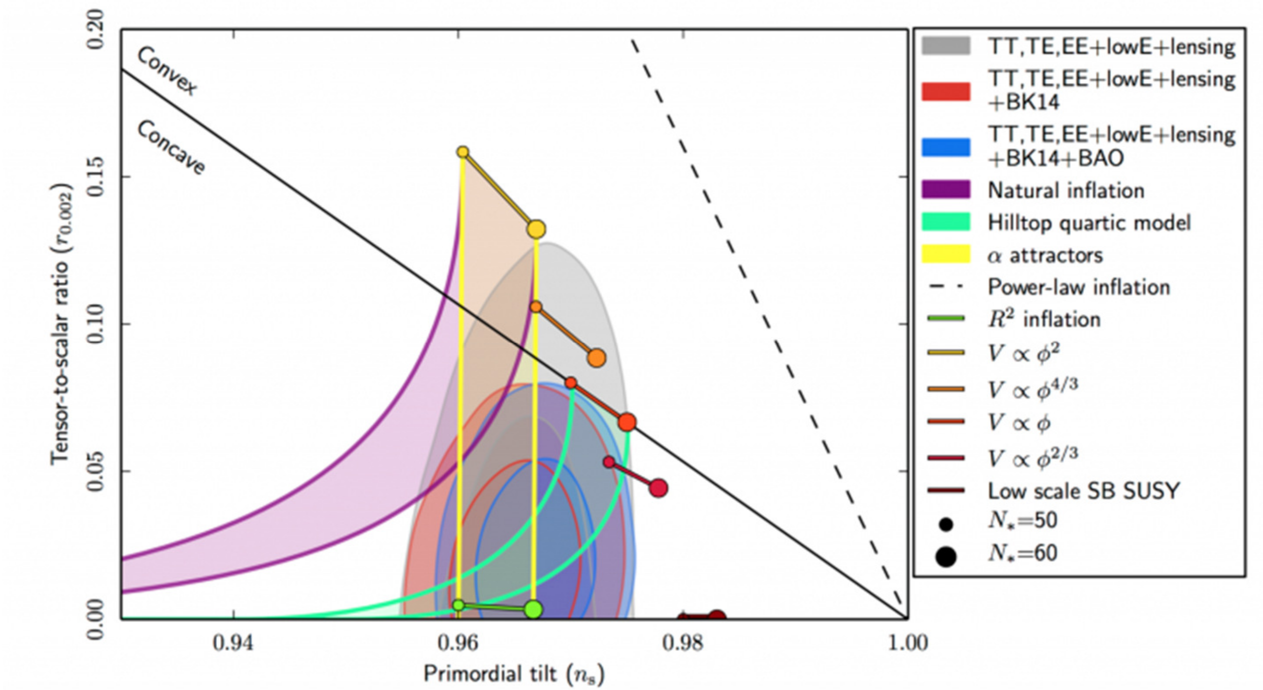


Inflation: scalar field

$$\epsilon = \frac{m_{\rm pl}^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 , \ \eta = \frac{m_{\rm pl}^2 V_{\phi\phi}}{8\pi V} , \ \xi^2 = \frac{m_{\rm pl}^4 V_\phi V_{\phi\phi\phi}}{64\pi^2 V^2}$$

$$n_s \approx 1 - 6\epsilon + 2\eta$$

$$r \approx 16\epsilon$$



Scalar-Tensor Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^{5} L_i$$

$$X = -\partial^{\mu}\phi\partial_{\mu}\phi/2$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X)\Box\phi$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_{\mu}\nabla_{\nu}\phi)(\nabla^{\mu}\nabla^{\nu}\phi)\right]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^{\mu}\nabla^{\nu}\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_{\mu}\nabla_{\nu}\phi)(\nabla^{\mu}\nabla^{\nu}\phi) + 2(\nabla^{\mu}\nabla_{\alpha}\phi)(\nabla^{\alpha}\nabla_{\beta}\phi)(\nabla^{\beta}\nabla_{\mu}\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

Horndeski Theories

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[G. Horndeski, Int. J. Theor. Phys. 10]

- Coincides with Generalized Galileon theories

$$\phi \to \phi + c, \quad \partial_\mu\phi \to \partial_\mu\phi + b_\mu$$

[Nicolis,Rattazzi,Trincherini, PRD 79]

Horndeski Cosmology (background)

- Field Equations: $L.H.S = R.H.S$
- In flat FRW:
- $$2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m$$
- $$K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4H\dot{X}G_{4,X} - 8\dot{H}XG_{4,X} - 8HX\dot{X}G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} + 2\left[2(\dot{H}X + H\dot{X}) + 3H^2X\right]G_{5,\phi} + 4HX\dot{\phi}G_{5,\phi\phi} = -p_m$$
- $$\frac{1}{a^3}\frac{d}{dt}(a^3J) = P_\phi$$

with $J = \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} + 2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X})$

$P_\phi = K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}$

[De Felice,Tsujikawa JCAP 1202]

Horndeski Cosmology (perturbations)

- Scalar perturbations: $ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$ $\quad \Rightarrow L.H.S = R.H.S$
- No-ghost condition: $Q_S \equiv \dfrac{w_1\left(4w_1w_3 + 9w_2^2\right)}{3w_2^2} > 0$
- No Laplacian instabilities condition: $c_S^2 \equiv \dfrac{3\left(2w_1^2w_2H - 4w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2\right) - 6w_1^2(\rho_m + p_m)}{w_1\left(4w_1w_3 + 9w_2^2\right)} > 0$

with $\quad w_1 \equiv 2\left(G_4 - 2XG_{4,X}\right) - 2X\left(G_{5,X}\dot{\phi}H - G_{5,\phi}\right)$

$$w_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4\left(\dot{\phi}G_{4,\phi X} - 4HG_{4,X}\right)X + 2G_{4,\phi}\dot{\phi}$$
$$+ 8X^2G_{5,\phi X}H + 2HX\left(6G_{5,\phi} - 5HG_{5,X}\dot{\phi}\right) - 4G_{5,XX}\dot{\phi}X^2H^2$$

$$w_3 \equiv 3X\left(K_{,X} + 2XK_{,XX}\right) + 6X\left(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\dot{\phi}HG_{3,X}\right)$$
$$+ 18H\left(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,X\phi X}\right)$$
$$+ 6H^2X\left(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi}\right)$$

$$w_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

[De Felice,Tsujikawa JCAP 1202]

Beyond Horndeski Theories

- Beyond Horndeski, free from Ostrogradski instabilities but still propagating 2+1 dof's:

$$L_{BH} = \sum_{i=2}^{5} L_i$$

$$L_2 = L_2^H[A_2]$$

$$L_3 = L_3^H[C_3 + 2XC_{3,X}] + L_2^H[XC_{3,\phi}]$$

$$L_4 = L_4^H[B_4] + L_3^H[C_4 + 2XC_{4,X}] + L_2^H[XC_{4,\phi}] - \frac{B_4 + A_4 - 2XB_{4,X}}{X^2} L^{gal\,1}$$

$$L_5 = L_5^H[G_4] + L_4^H[C_5] + L_3^H[D_5 + 2XD_{5,X}] + L_2^H[XD_{5,\phi}] + \frac{XB_{5,X} + 3A_5}{3(-X)^{5/2}} L^{gal\,2}$$

$$X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$A_i = A_i(\phi, X)$$

$$B_i = B_i(\phi, X)$$

with

$$L^{gal\,1} = X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] - 2\left[(\nabla^\mu\phi\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)(\Box\phi) - (\nabla^\mu\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla_\lambda\phi)(\nabla^\lambda\nabla^\nu\phi)\right]$$

$$L^{gal\,2} = X\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\nu\nabla^\rho\phi)(\nabla^\mu\nabla_\rho\phi)\right]$$

$$- 3\left[\begin{array}{l}(\Box\phi)^2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\phi) - 2(\Box\phi)(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\phi) \\ -(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi) + 2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi)\end{array}\right]$$

$$C_3 = \frac{1}{2}\int A_3(-X)^{-3/2}dX \qquad C_5 = -\frac{1}{4}X\int B_{5,\phi}(-X)^{-3/2}dX$$

$$C_4 = -\int B_{4,\phi}(-X)^{-1/2}dX \qquad D_5 = -\int C_{5,\phi}(-X)^{-1/2}dX \qquad G_5 = -\int B_{5,X}(-X)^{-1/2}dX$$

- Primary constraint prevents the propagation of extra degrees of freedom

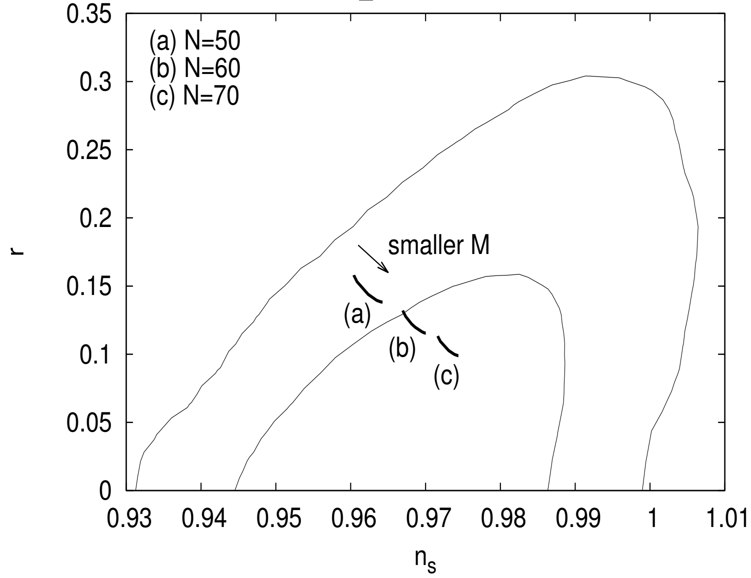
[Gleyzes,Langlois,Piazza,Vernizzi, PRL 114], [Crisostomi,Hull,Koyama,Tasinato, JCAP 1603]

Inflation in Horndeski Theories

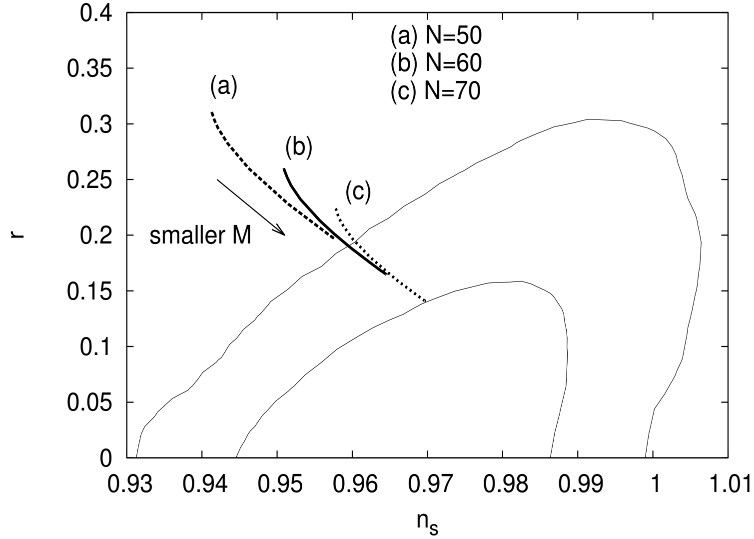
$$K(\phi, X) = X - V(\phi), \quad G_3(\phi, X) = \frac{c_3}{M^3} X, \quad G_4 = G_5 = 0$$

[Ohashi,Tsujikawa, JCAP 1210]

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$



$$V(\phi) = \frac{1}{4} \lambda \phi^4$$



Inflation in Horndeski Theories

- $K(\phi, X) = X - V(\phi), \quad G_3(\phi, X) = \frac{c_3}{M^3} X, \quad G_4 = G_5 = 0$ [Ohashi,Tsujikawa, JCAP 1210]

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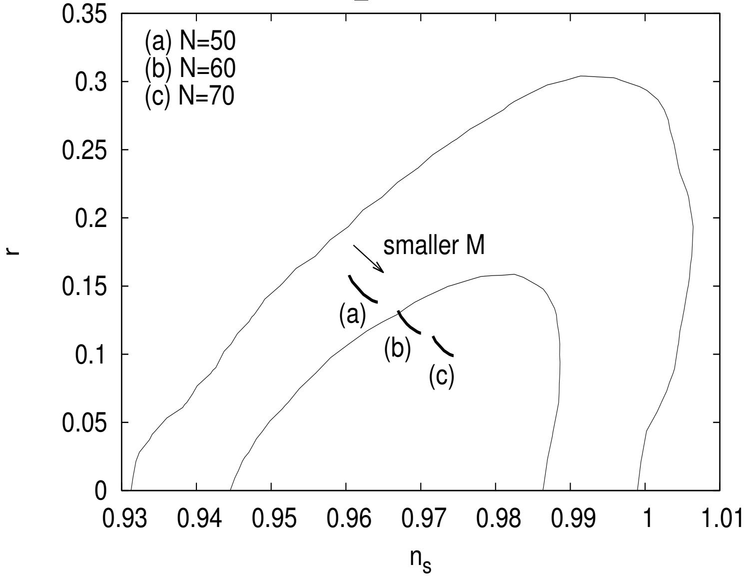
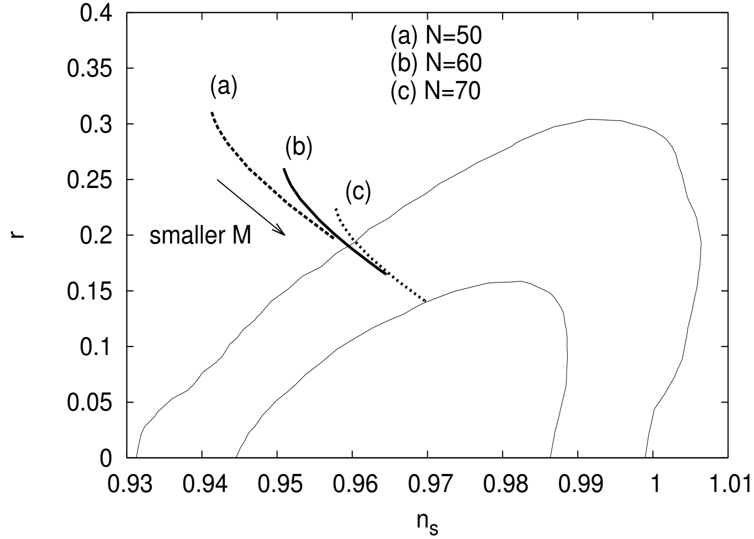


$V(\phi) = \frac{1}{4} \lambda \phi^4$



- G-Inflation (Shift-symmetric): $K(\phi, X) = X + \frac{X^2}{2M^3\mu}, \quad G_3(\phi, X) = \frac{1}{M^3} X, \quad G_4 = G_5 = 0$

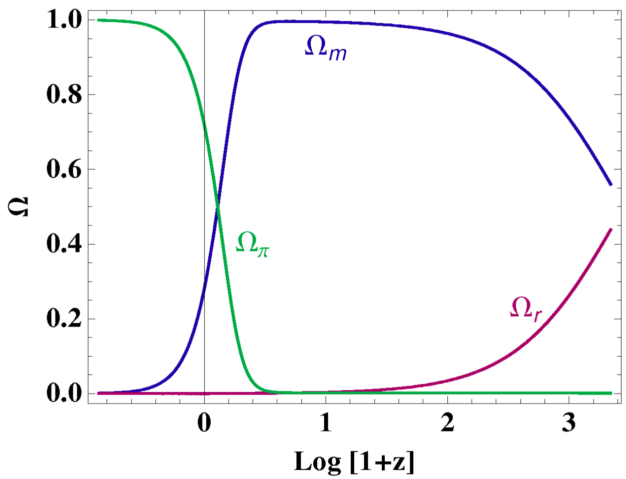
$r \approx 0.17$

[Kobayashi,Yamaguchi,Yokoyama PRL 105] [Banerjee, Saridakis PRD 95]

Dark Energy in Horndeski Theories

- $K(\phi, X) = c_2 X\,,\;\; G_3(\phi, X) = c_3\,,\;\; G_4 = 1,\;\; G_5 = c_5$

- Background evolution: Universe thermal history



[Ali,Gannouji,Sami PRD 82] [Leon, Saridakis JCAP 1303]

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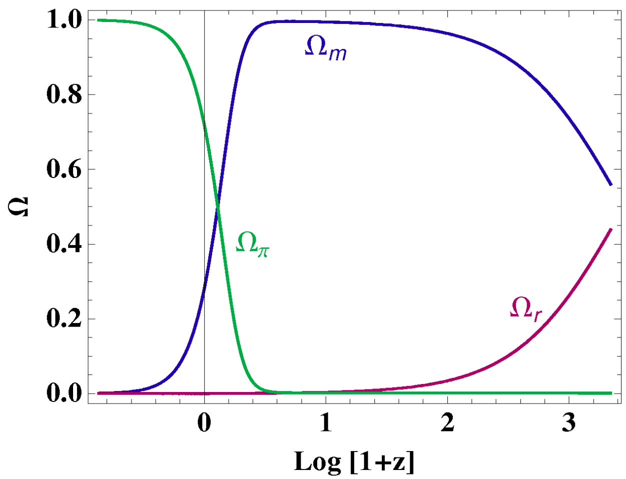
[Leon, Saridakis JCAP 1303]

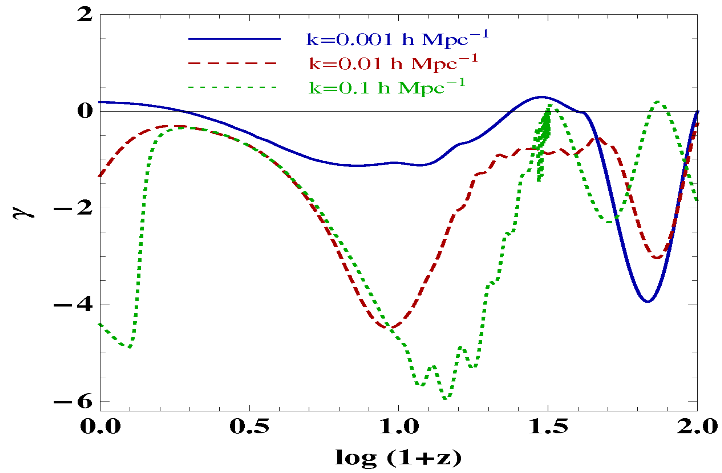


- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\rho_m\delta_m$

  with $G_{eff} = G_{eff}(\phi, K, G_3, G_4, G_5)$

- Clustering growth rate: $\dfrac{d\ln\delta_m}{d\ln a} = \Omega_m^{\gamma}(a)$

  γ(z): Growth index.



[Ali,Gannouji,Sami PRD 82]

f(R) gravity

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R)+S_m\left(g_{\mu\nu},\psi\right)$$

$$f'(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\left[\nabla_\mu\nabla_\mu-g_{\mu\nu}\Diamond\right]f'(R)=8\pi G\,T_{\mu\nu}$$

- Field Equations (metric formalism):

- Conformal transformation: $g_{\mu\nu}\to\tilde{g}_{\mu\nu}=f'(R)g_{\mu\nu}\equiv\phi g_{\mu\nu},\ d\varphi=\sqrt{\frac{2\omega_0+3}{16\pi G}}\frac{d\phi}{\phi}$

$$\Rightarrow_{\omega_0=0}\quad S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\pi G}-\frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi-U(\varphi)\right]+S_m\left(e^{-\sqrt{16\pi G/3}}\tilde{g}_{\mu\nu},\psi\right)\qquad U(\varphi)=\frac{Rf'(R)-f(R)}{16\pi G\left[f'(R)\right]^2}$$

[Capozziello, De Laurentis, Phys. Rept. 509]

f(R) cosmology - Inflation

- Firedmann Equations (metric formalism):

$$3FH^2 = \frac{FR-f}{2} - 3H\dot{F} + 8\pi G\,\rho_m$$

$$-2F\dot{H} = \ddot{F} - H\dot{F} + 8\pi G\left(\rho_m + p_m\right)$$

$$F(R) \equiv f'(R)$$

$$R = 12H^2 + 6\dot{H}$$

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$$F(R) \equiv f'(R)$$

$$R = 12H^2 + 6\dot{H}$$

- Inflation: e.g. Starobinsky inflation

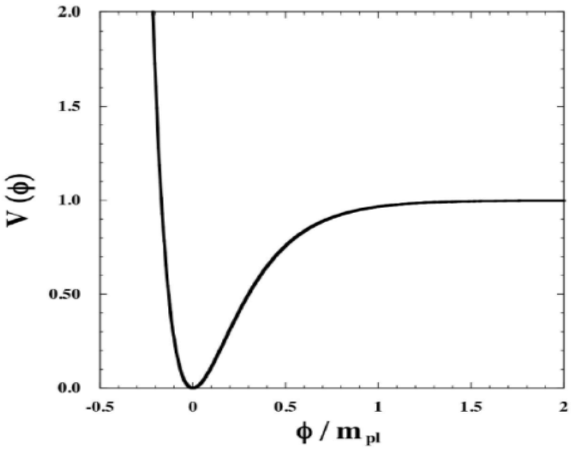
$$f(R) = R + \frac{R^2}{6M^2}$$

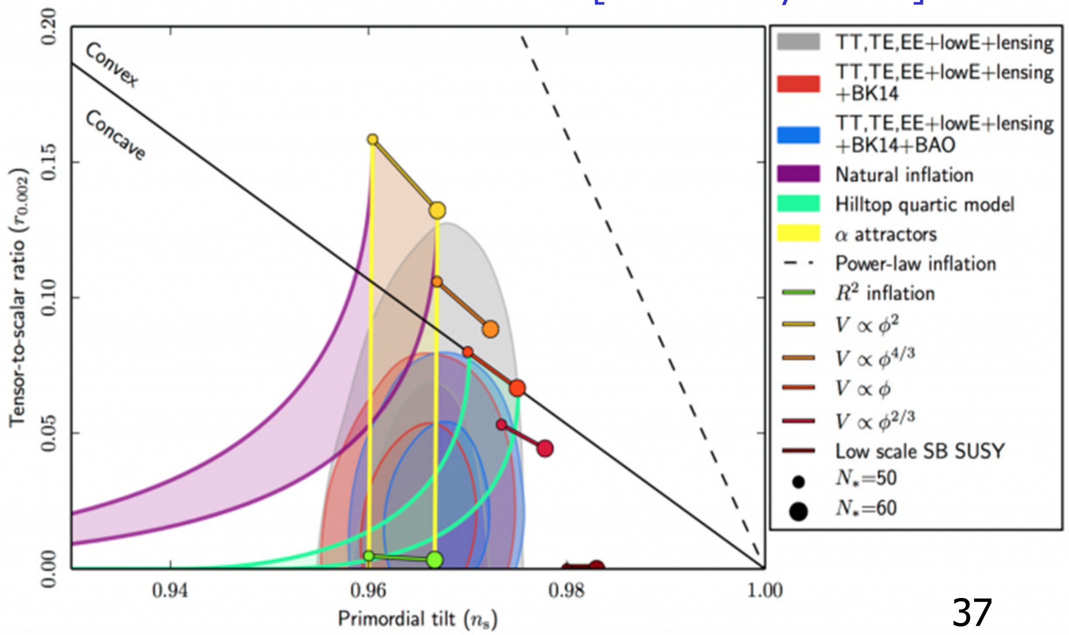
$$\Rightarrow V(\phi) = \frac{3M^2}{32\pi G}\left(1 - e^{-\sqrt{2/3}8\pi G\phi}\right)$$

$$H \approx H_i - \frac{M^2}{6}(t - t_i)$$

$$T_{reh} \le 3\times10^{17} g_*^{1/4} \left(\frac{M}{m}\right)^{3/2} GeV \qquad M \approx 3\times10^{13}\ GeV$$

[Starobinsky PL 91]





f(R) cosmology – Dark energy

$$8\pi G\,\rho_{DE} = \frac{FR - f}{2} - 3H\dot{F} + 3H^2(1-F)$$

$$8\pi G\,p_{DE} = \ddot{F} + 2H\dot{F} - \frac{FR - f}{2} - \left(3H^2 + 2\dot{H}\right)(1-F)$$

for viable: $f_{,R} > 0,\ f_{,RR} > 0,\ for\ R \geq R_0 (> 0)$

[Starobinsky PLB 91]

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$$8\pi G\,\rho_{DE}=\frac{FR-f}{2}-3H\dot F+3H^2(1-F)$$

for viable: $f_{,R}>0,\ f_{,RR}>0,\ for\ \ R\geq R_0(>0)$

$$8\pi G\,p_{DE}=\ddot F+2H\dot F-\frac{FR-f}{2}-\left(3H^2+2\dot H\right)(1-F)$$

[Starobinsky PLB 91]

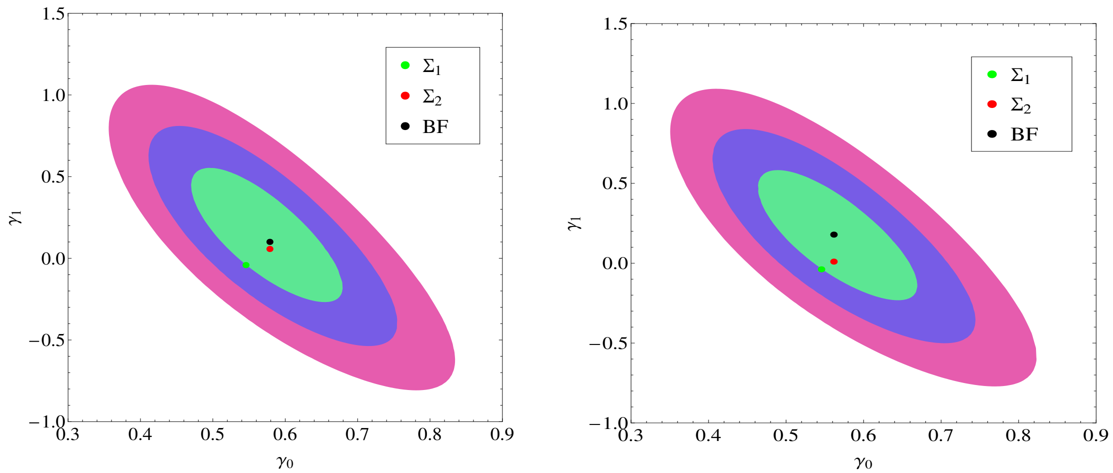
model	$f(R)$	Constant parameters
(i) Hu-Sawicki	$R-\frac{c_1 R_{\rm HS}(R/R_{\rm HS})^p}{c_2(R/R_{\rm HS})^p+1}$	$c_1,\ c_2,\ p(>0), R_{\rm HS}(>0)$
(ii) Starobinsky	$R+\lambda R_{\rm S}\left[\left(1+\frac{R^2}{R_{\rm S}^2}\right)^{-n}-1\right]$	$\lambda(>0),\ n(>0),\ R_{\rm S}$
(iii) Tsujikawa	$R-\mu R_{\rm T}\tanh\left(\frac{R}{R_{\rm T}}\right)$	$\mu(>0),\ R_{\rm T}(>0)$
(iv) Exponential	$R-\beta R_{\rm E}\left(1-e^{-R/R_{\rm E}}\right)$	$\beta,\ R_{\rm E}$

[Bamba,Geng,Lee JCAP 1011]

- $$\ddot\delta_m+2H\dot\delta_m=4\pi G_{eff}\,\rho_m\delta_m$$

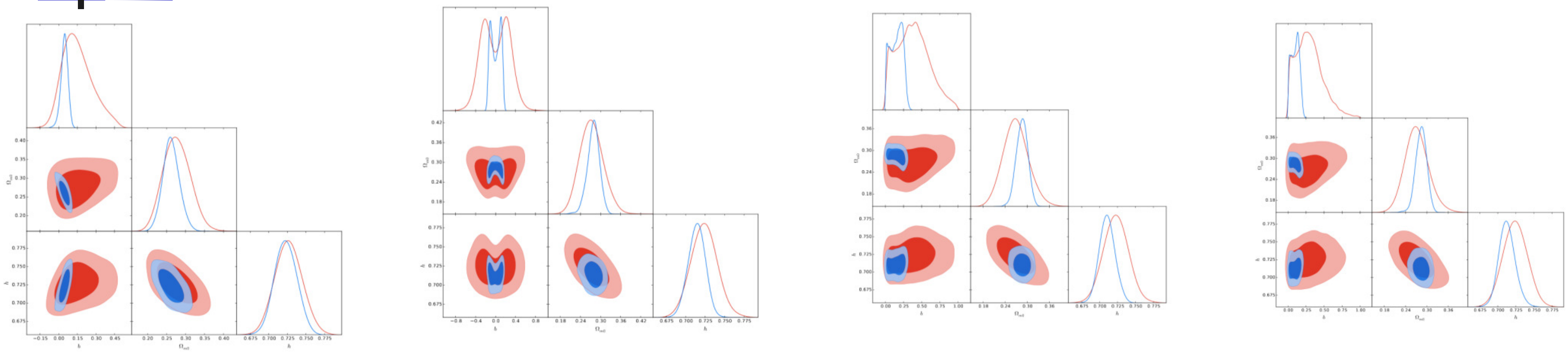
$$G_{eff}=\frac{G}{f'}\,\frac{1+4\frac{k^2}{a^2}\frac{f''}{f'}}{1+3\frac{k^2}{a^2}\frac{f''}{f'}}$$

$$\frac{d\ \ln\delta_m}{d\ \ln a}=\Omega_m^{\gamma}(a)$$



[Basilakos,Nesseris,Perivolaropoulos PRD 87]

f(R) cosmology – Dark energy



Models	CC+ H_0				JLA + BAO + CC + H_0			
	AIC	ΔAIC	BIC	ΔBIC	AIC	ΔAIC	BIC	ΔBIC
ΛCDM Model	28.205	0	36.809	0	721.084	0	749.017	0
Hu-Sawicki Model	28.744	0.539	38.782	1.973	720.840	−0.244	753.428	4.411
Starobinsky Model	29.096	0.891	39.134	2.325	721.726	0.642	754.314	5.297
Tsujikawa Model	29.407	1.202	39.445	2.636	722.966	1.882	755.554	6.537
Exponential Model	29.310	1.105	39.347	2.538	722.548	1.464	755.136	6.119

[Nunes, Pan, Saridakis, Abreu JCAP 1701]

Bi-scalar Theories

- Modified gravity propagating 2+2 dof's $S=\int d^4x\sqrt{-g}\,f\left(R,(\nabla R)^2,\Diamond R\right)$

- For $f\left(R,(\nabla R)^2,\Diamond R\right)=K\left(R,(\nabla R)^2\right)+Q\left(R,(\nabla R)^2\right)\Diamond R$ [Naruko,Yoshida,Mukohyama CQG 33]

$$\Rightarrow S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\hat{R}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{\sqrt{6}}e^{-\sqrt{2/3}\chi}\hat{g}^{\mu\nu}Q\partial_\mu\chi\partial_\nu\phi+\frac{1}{4}e^{-2\sqrt{2/3}\chi}K+\frac{1}{2}e^{-\sqrt{2/3}\chi}Q\hat{\Diamond}\phi-\frac{1}{4}e^{-\sqrt{2/3}\chi}\phi\right]$$

$$K=K(\phi,B),\;\; G=G(\phi,B),\;\; B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

Bi-scalar Theories

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[Naruko,Yoshida,Mukohyama CQG 33]

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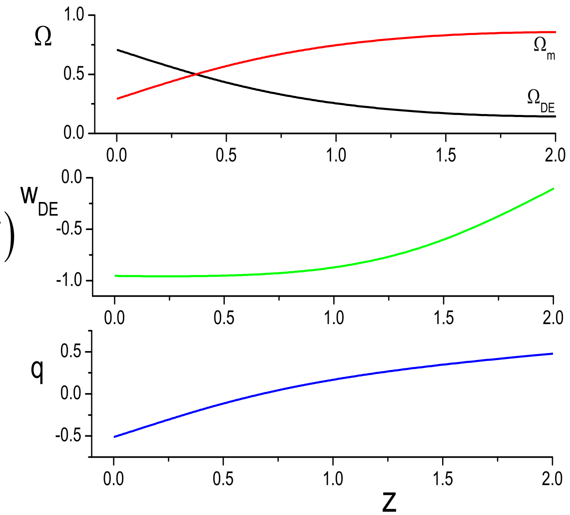
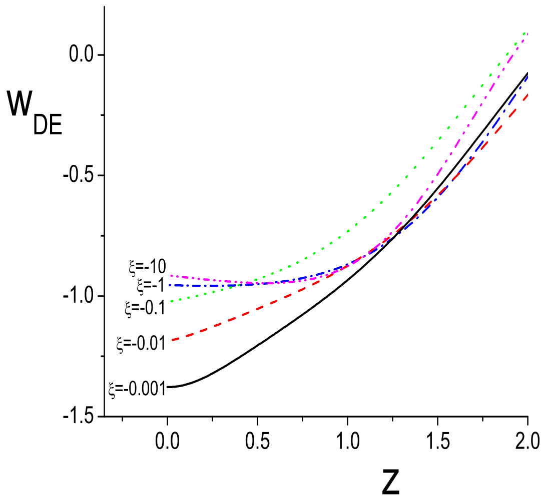
$$K=K(\phi,B),\ G=G(\phi,B),\ B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

- eg.: $K(\phi,B)=\frac{\phi}{2},\ G(\phi,B)=\xi B$

$$\rho_{DE}=\frac{1}{2}\dot{\chi}^2-\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\xi\dot{\phi}^3\left(\sqrt{6}\dot{\chi}-6H\right)$$

$$p_{DE}=\frac{1}{2}\dot{\chi}^2+\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\frac{1}{3}\xi\dot{\phi}^2\left(\sqrt{6}\dot{\phi}\dot{\chi}+6\ddot{\phi}\right)$$



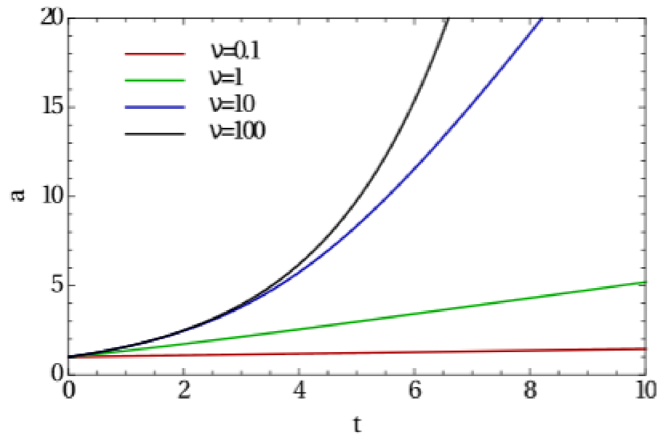


[Saridakis,Tsoukalas PRD 93]

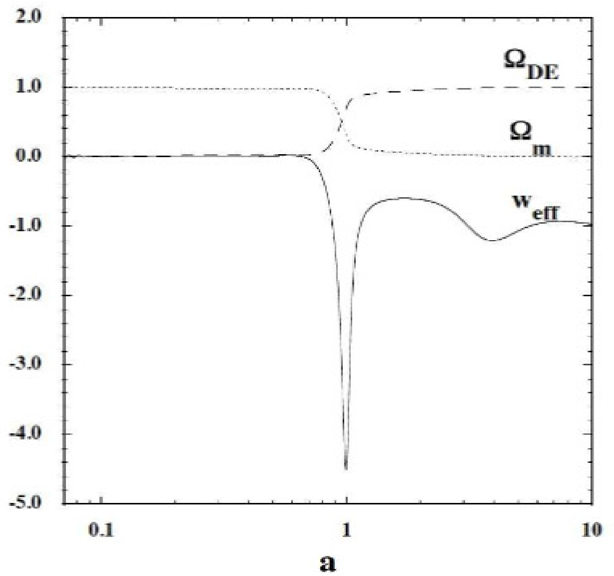
f(G) Theories

- Gauss-Bonnet Invariant: $\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}R + f(\mathcal{G})\right] + S_m(g_{\mu\nu}, \Psi_m)$$



[Kanti, Gannouji, Dadhich PRD 92]

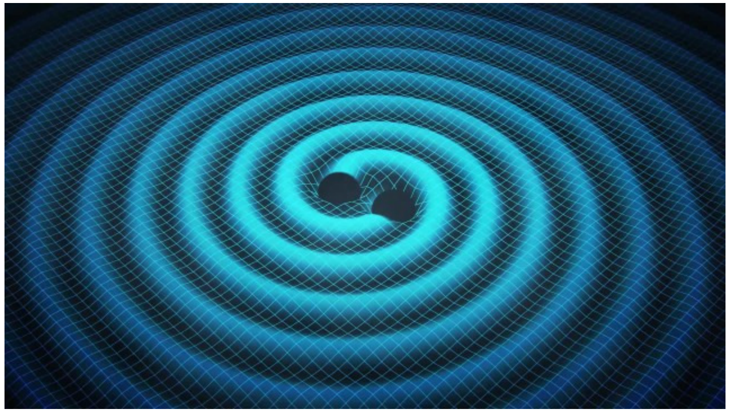


[Nojiri, Odintsov., Sasaki PRD 71]
[De Felice, Tsujikawa PLB 675]

Gravitational waves

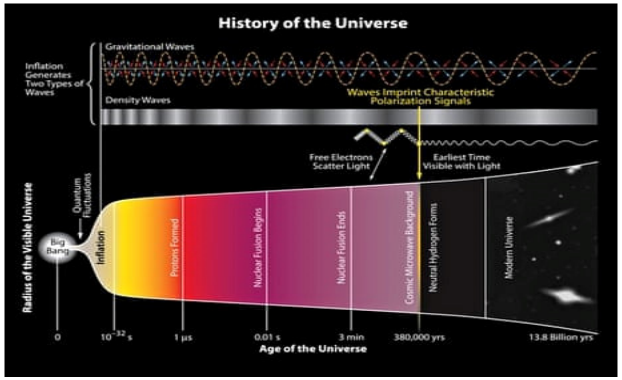
- The GWs are the tensor perturbations of the metric. Predicted in 1915, first observed in 2015. First astronomical observation ever, not related to E/M (or neutrinos).

- GWs from mergers:



[Abbott et al, LIGO Virgo PRL 116]

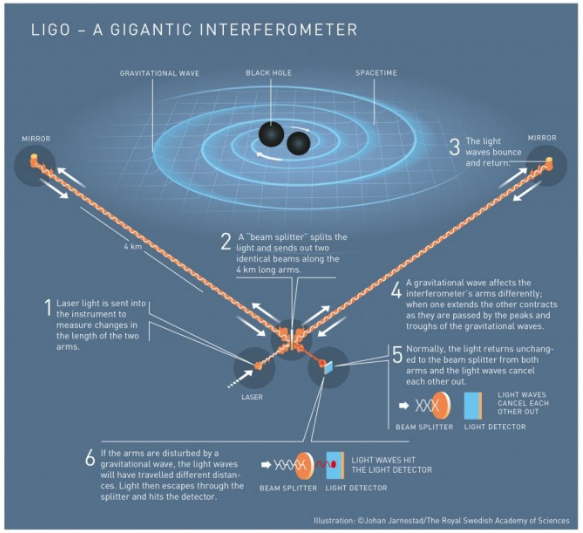
- Primordial GWs:

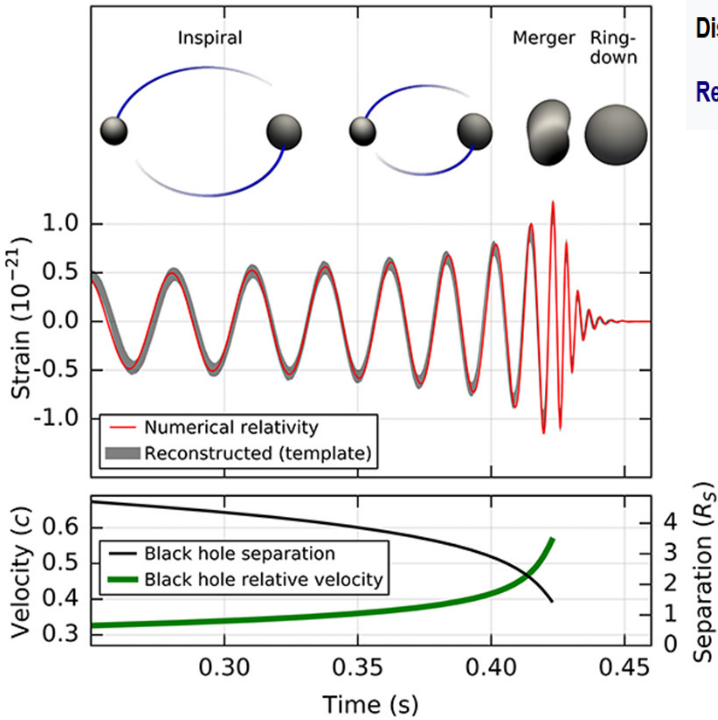


Gravitational waves

- GW150914: Two black holes with 36^{+5}_{-4} M⊙ and 29^{+4}_{-4} M⊙, resulting in a 62^{+4}_{-4} M⊙ black hole

Louisiana.
Washington
4km
10^{-18}m





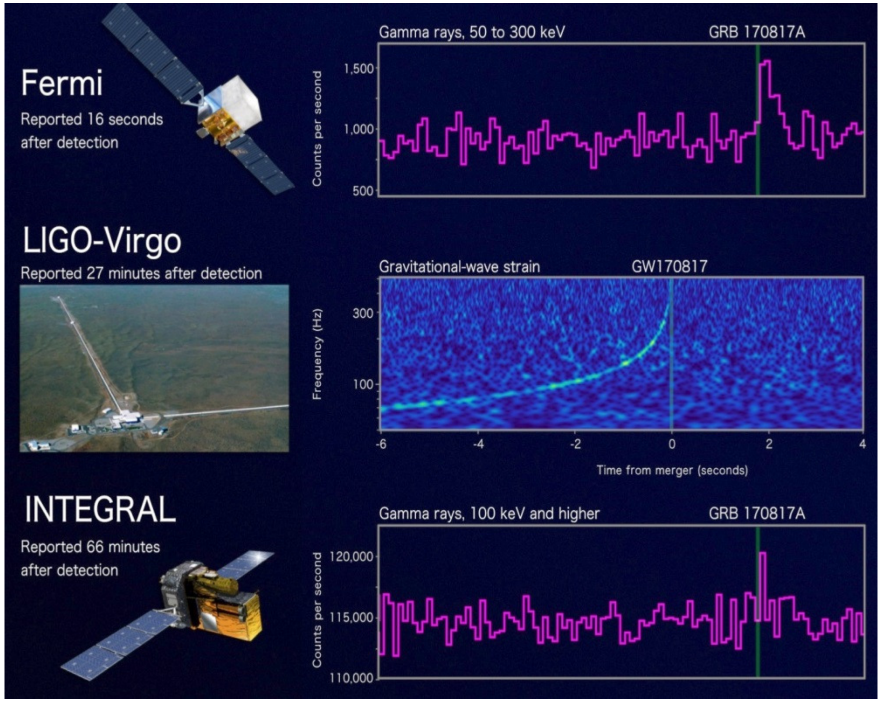
Distance	410^{+160}_{-180} Mpc
Redshift	$0.093^{+0.030}_{-0.036}$

[Abbott et al, LIGO Virgo PRL 116]

2017 Nobel Price in Physics

Gravitational waves

- GW170817: Two neutron stars, distance 40 Mpc, redshift 0.0099
- GRB170817A: The Electromagnetic counterpart.

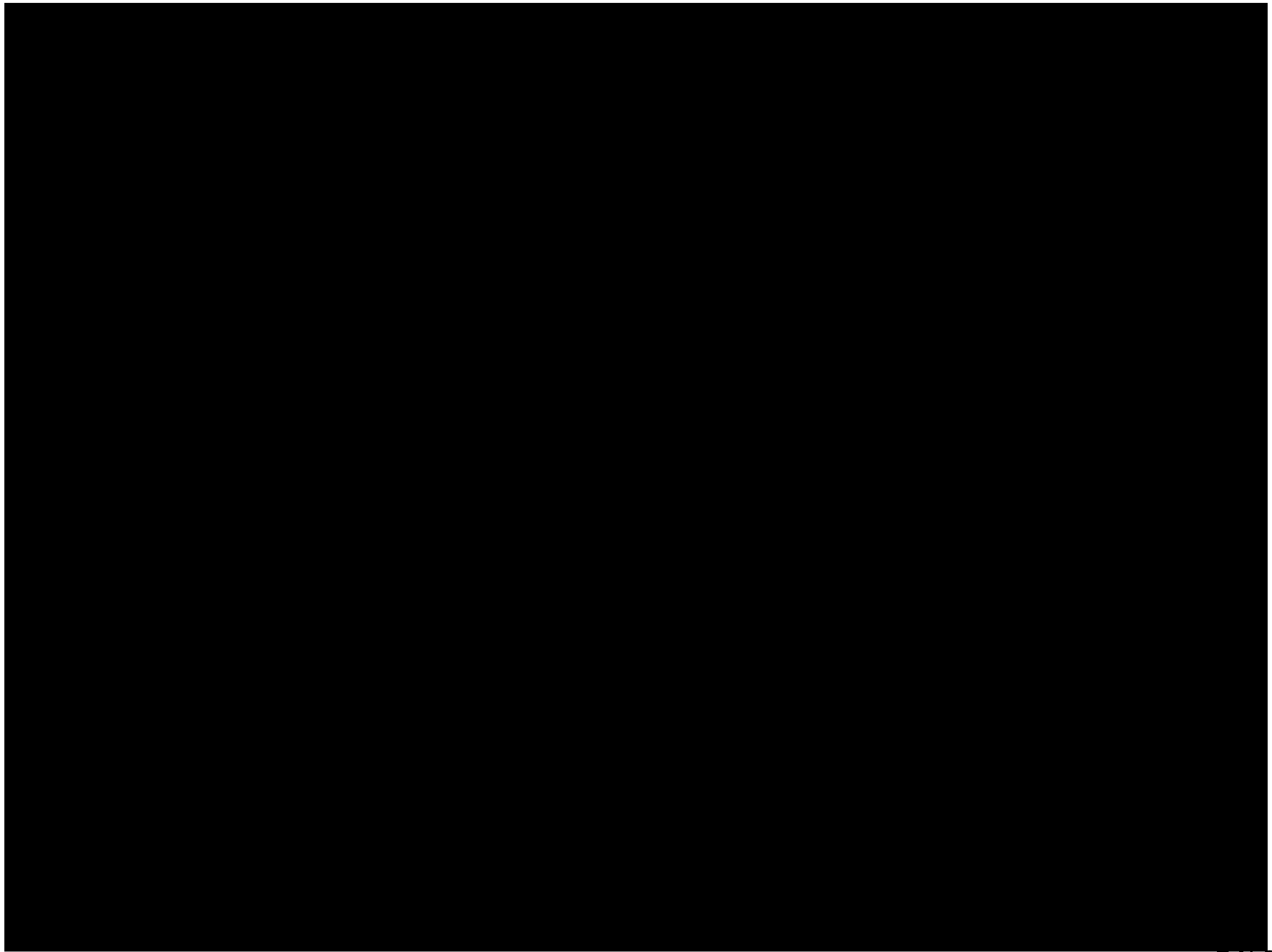


[Goldstein et al, Fermi Gamma Ray Burst Monitor Astrophys.J 848]

[Abbott et al, LIGO Virgo PRL 119]

- The era of multi-messenger astronomy begins!

Multi-messenger astronomy



Gravitational waves

- In case of GWs from black hole mergers we know their properties at the moment of detection, and their direction (in case of three detectors). Assuming GR and ΛCDM we can extract their speed, distance, and properties at the moment of emission.

Gravitational waves

- In case of GWs from black hole mergers we know their properties at the moment of detection, and their direction (in case of three detectors). Assuming GR and ΛCDM we can extract their speed, distance, and properties at the moment of emission.

- In case of GWs from neutron star mergers, and their E/M counterpart, we know their properties at the moment of detection and their direction, but using the implied physics from the E/M information we can extract their speed, distance and properties at the moment of emission, independently of the underlying gravitational theory and cosmological scenario.

- Great tool for testing General Relativity and cosmological scenarios!

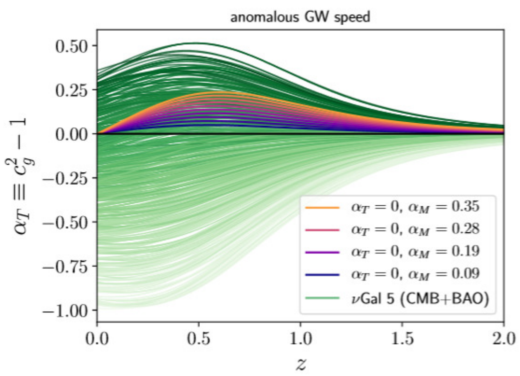
Gravitational waves

- An immediate result: The speed of GWs is equal to the speed of light!

  GW170817 time delay $1.74 \pm 0.05\text{s}$ constrains: $-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$

- Excludes a large number of theories that were consistent with other data!

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)$·Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817



[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves

- For tensor perturbations:

$$g_{00} = -1\ , \quad g_{0i} = 0\ ,$$
$$g_{ij} = a^2(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj})$$

$$\ddot{h}_{ij} + (3 + \alpha_M)\dot{h}_{ij} + (1 + \alpha_T)\frac{k^2}{a^2}h_{ij} = 0$$

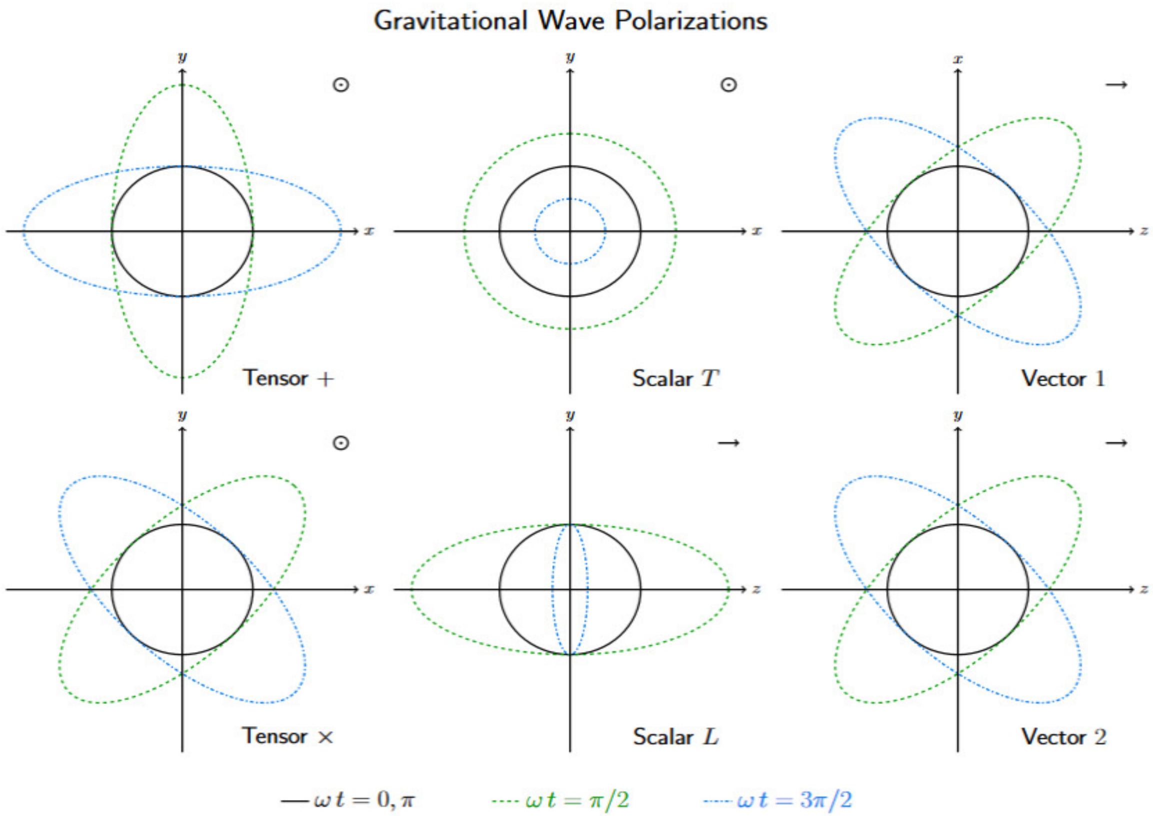
$$\alpha_M = \frac{d\log(M_*^2)}{d\log a} \qquad c_g^2 = (1 + \alpha_T)$$

- $$h_{\rm GW} \sim h_{\rm GR} \underbrace{e^{-\frac{1}{2}\int \nu \mathcal{H} d\eta}}_{\text{Affects amplitude}} \underbrace{e^{ik\int(\alpha_T + a^2m^2/k^2)^{1/2}d\eta}}_{\text{Affects phase}}$$

[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves

- Polarizations:



[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves in modified gravity

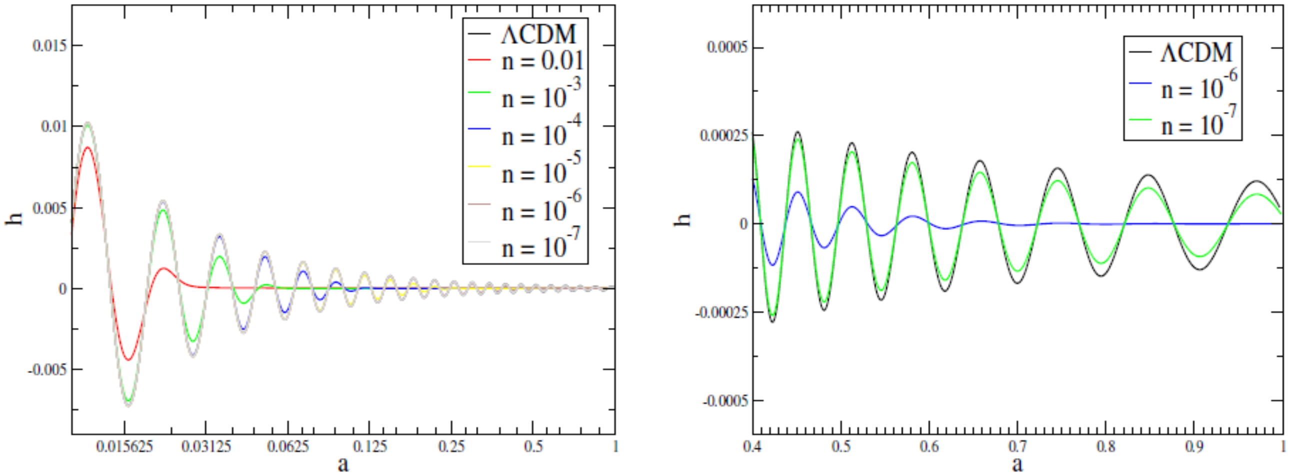
- Gw's propagation at cosmological scales: $h = e^{-\mathcal{D}} e^{-ik\Delta T} h_{GR}$

$\mathcal{D} = \frac{1}{2}\int \nu \mathcal{H} d\tau'$ (affects amplitude) $\Delta T = \int \left(1 - c_T - \frac{a^2\mu^2}{2k^2}\right) d\tau'$ (affects phase)

- In f(T) gravity:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

$$\beta_T \equiv -\frac{\dot{f}_T}{3H f_T}$$



[Cai, Li, Saridakis, Xue PRD 97]

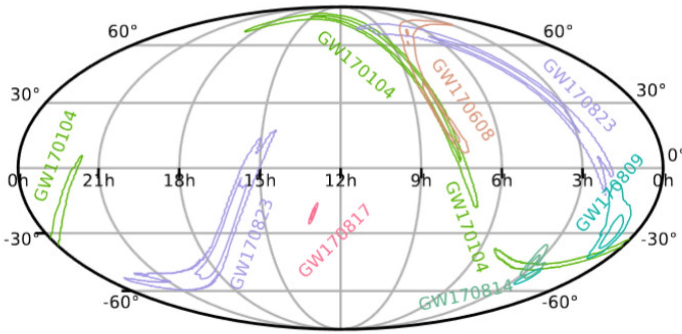
[Farrugia, Said, Gakis, Saridakis, PRD 97]

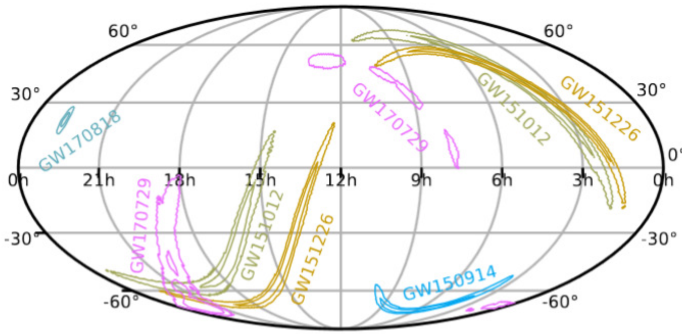
[Soudi, Farrugia, Gakis, Said, Saridakis, PRD 100]

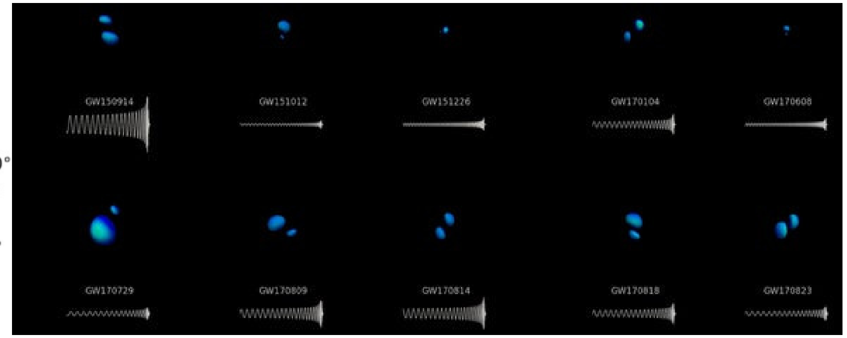
[Nunes, Pan, Saridakis, PRD98]

Observations: Present-Future

- Observations: 43 up to now (30 BH-BH, 4 NS-NS, 4 NS-BH, 4 uncertain, 19-85 Msun, 320-2800 MPc)







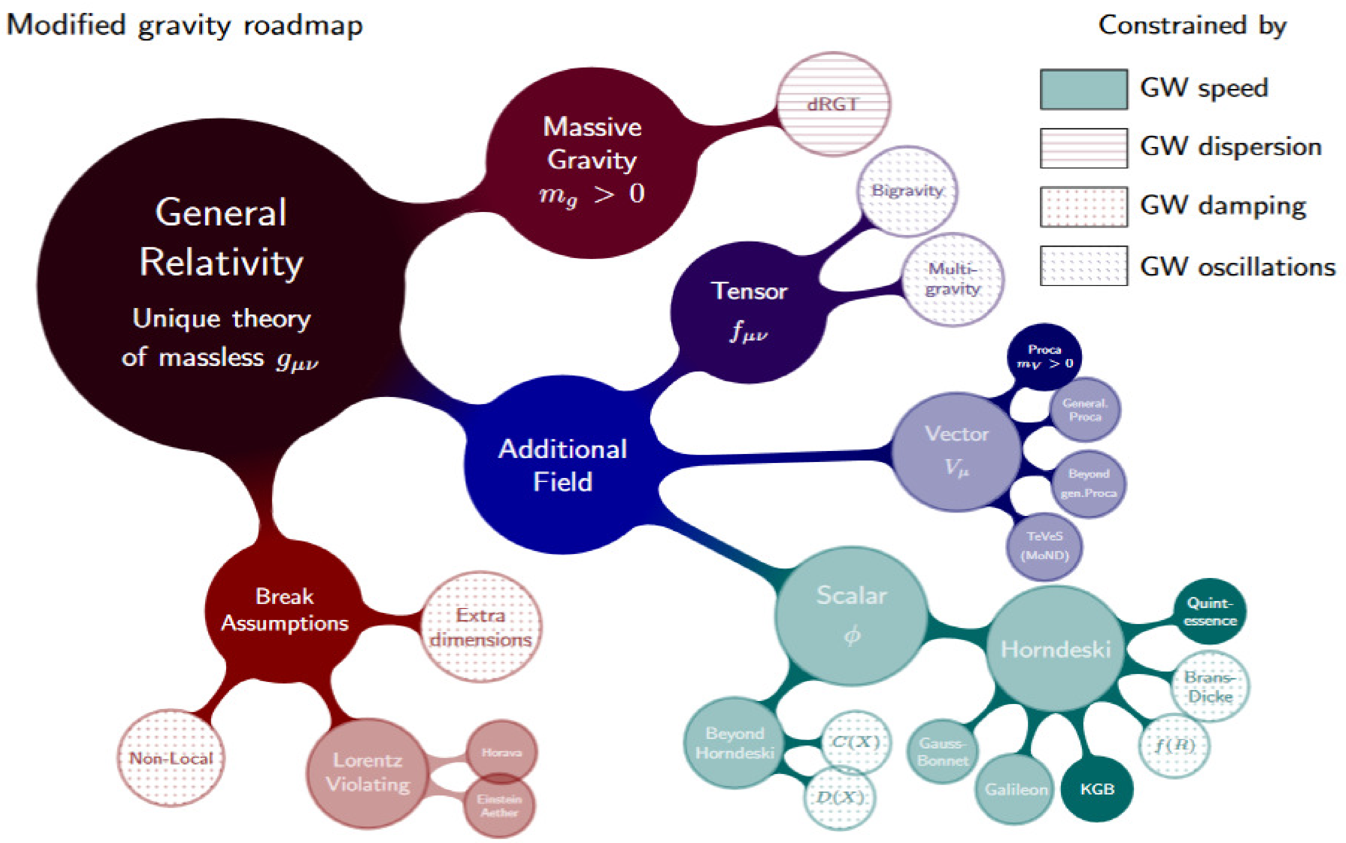
Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$

Designation
150914+09:50:45UTC
151226+03:38:53UTC
151012+09:54:43UTC
151019+00:23:16UTC
150928+10:49:00UTC
151218+18:30:58UTC
160103+05:48:36UTC
151202+01:18:13UTC
160104+03:51:51UTC
151213+00:12:20UTC
150923+07:10:59UTC
151029+13:34:39UTC
151206+14:19:29UTC
151202+15:32:09UTC
151012+06:30:45UTC
151116+22:41:48UTC
151121+03:34:09UTC
150922+05:41:08UTC
151008+14:09:17UTC
151127+02:00:30UTC

[LIGO-Virgo Collaborations 1811.12907]

- Expectations: Many thousands in the next years

Gravitational waves and Modified Gravity



[Ezquiaga, Zumalacarregui PRL 119]

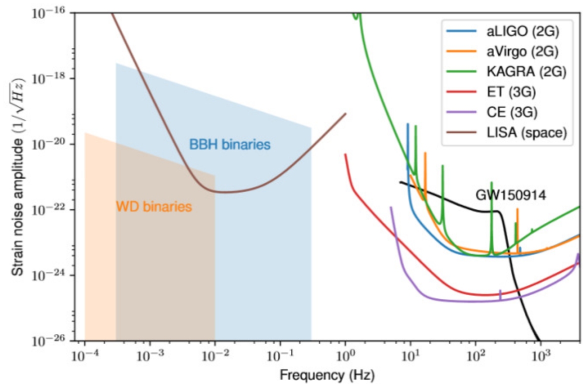
Conclusions

- i) The Standard Model of Cosmology may ask for new physics, definitely for inflation and dark matter, probably for dark energy.

- ii) We can modify the Universe content, or/and the gravitational theory. Torsional gravity is a good candidate.

- iii) We use various observational data (SnIa, CMB, BAO, H(z), LSS etc) in order to constrain the proposed theories.

- iv) The advancing gravitational wave astronomy, and especially multi-messenger astronomy offers a novel tool to test General Relativity and cosmological scenarios in great accuracy.



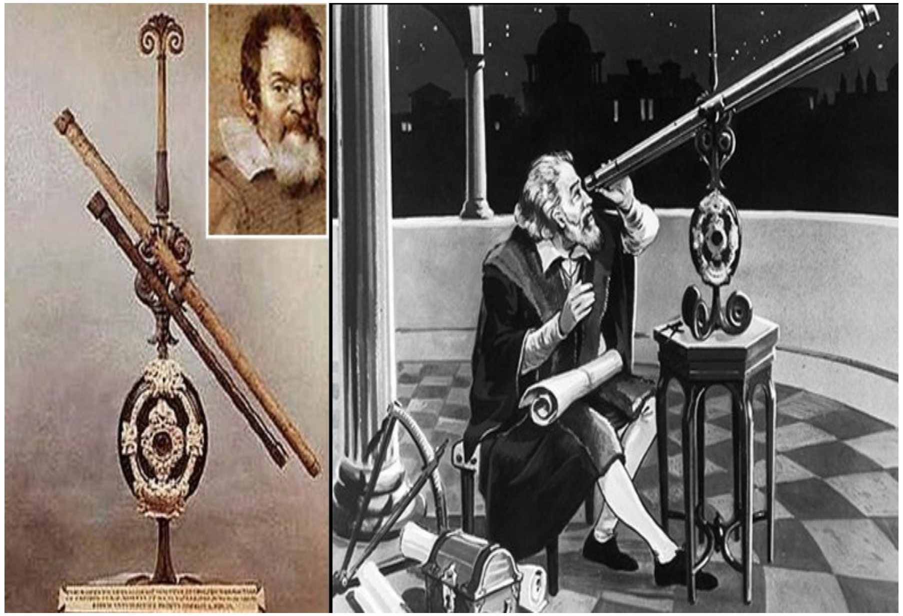
- v) A new era has begun!

Outlook

- A huge project is ahead for the community:
- i) Calculate the exact form of GWs created from mergers in various gravitational theories (needs numerical gravity).
- ii) Calculate the propagation of these GWs from emission to detection for various cosmological scenarios.
- iii) Use multi-messenger data to test General Relativity, break degeneracies and constrain or exclude the various theories.
- iv) Elaborate also the creation and possible detection of primordial GWs.
- v) For f(T) gravity, f(R,G), running vacuum, higher-order theories, f(T,TG) gravity, f(Q) gravity, etc, currently under investigation

  [Saridakis, Capozziello, Basilakos, Said, Cai, Marciano, Modesto, Nunes]
- vi) Get prepared for the huge flow of data that will come!

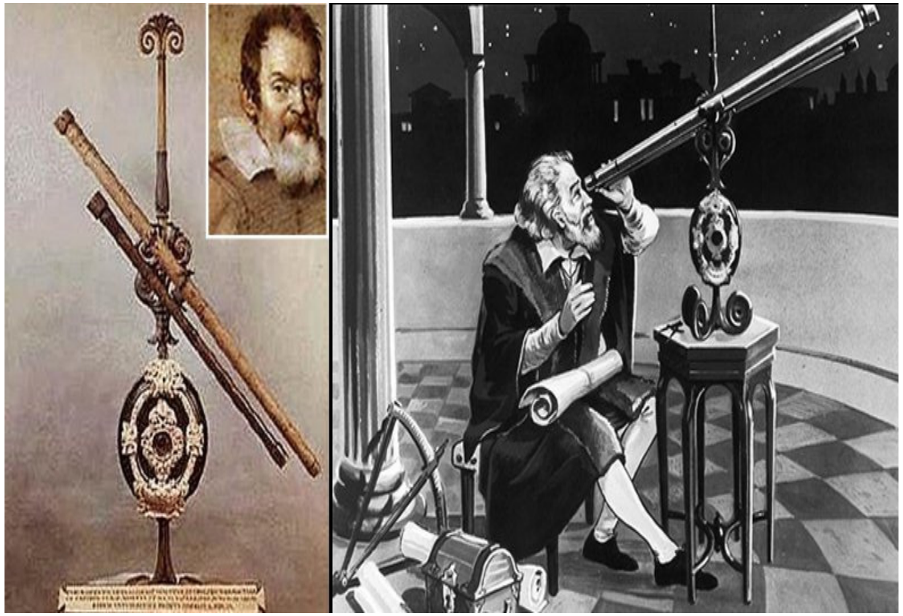
Multi-messenger Astronomy Era!



EM observations: 400 years

GW observations: 4 years

Multi-messenger Astronomy Era!



EM observations: 400 years

GW observations: 4 years

THANK YOU!

Gravitational waves in f(T) gravity

- For tensor perturbations: $g_{00} = -1\ , \quad g_{0i} = 0\ ,$

$$g_{ij} = a^2\Big(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\Big)$$

i.e.

$$\bar{e}^0_\mu = \delta^0_\mu\ ,$$
$$\bar{e}^a_\mu = a\delta^a_\mu + \frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij} + \frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ ,$$
$$\bar{e}^\mu_0 = \delta^\mu_0\ ,$$
$$\bar{e}^\mu_a = \frac{1}{a}\delta^\mu_a - \frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij} + \frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}$$

- We obtain:

$$^{(3)}R \approx -\frac{1}{4}a^{-2}(\partial_i h_{kl}\partial_i h_{kl})\ ,$$
$$K^{ij}K_{ij} \approx 3H^2 + \frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$
$$K \approx 3H\ ,$$
$$\boxed{T = T^{(0)} + O(h^2) = 6H^2 + O(h^2)}$$

- And finally:

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla}h_{ij}\cdot\vec{\nabla}h_{ij} - \dot{h}_{ij}\dot{h}_{ij}\big) + 6H^2 f_T - 12H\dot{f}_T - T^{(0)}f_T + f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97] [Li, Cai, Cai, Saridakis, JCAP 1810]

Gravitational waves in f(T) gravity

- Varying the action and going to Fourier space we get the equation for GWs:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T}$

$$h^{(1)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\gamma_1^{(1)1} & {B_1}^2\exp(ip_\mu x^\mu) & 0 \\ 0 & {B_1}^2\exp(ip_\mu x^\mu) & -2\gamma_1^{(1)1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

- An immediate result: The speed of GWs is equal to the speed of light!
- GW170817 constraints that

$$|c_g/c - 1| \leq 4.5 \times 10^{-16}$$

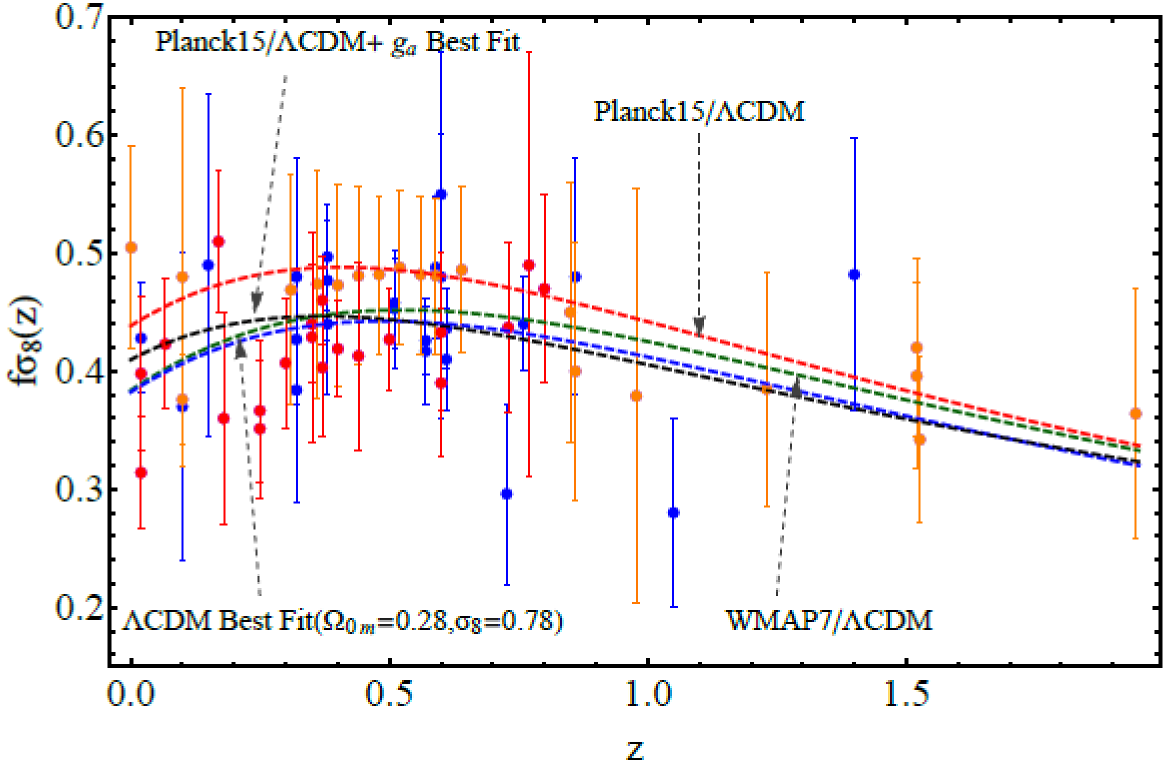
are trivially satisfied.

[Cai, Li, Saridakis, Xue, PRD 97]

Tension1 – fσ8

- Tension between the data and Planck/ΛCDM. The data indicate a lack of "gravitational power" in structures on intermediate-small cosmological scales.

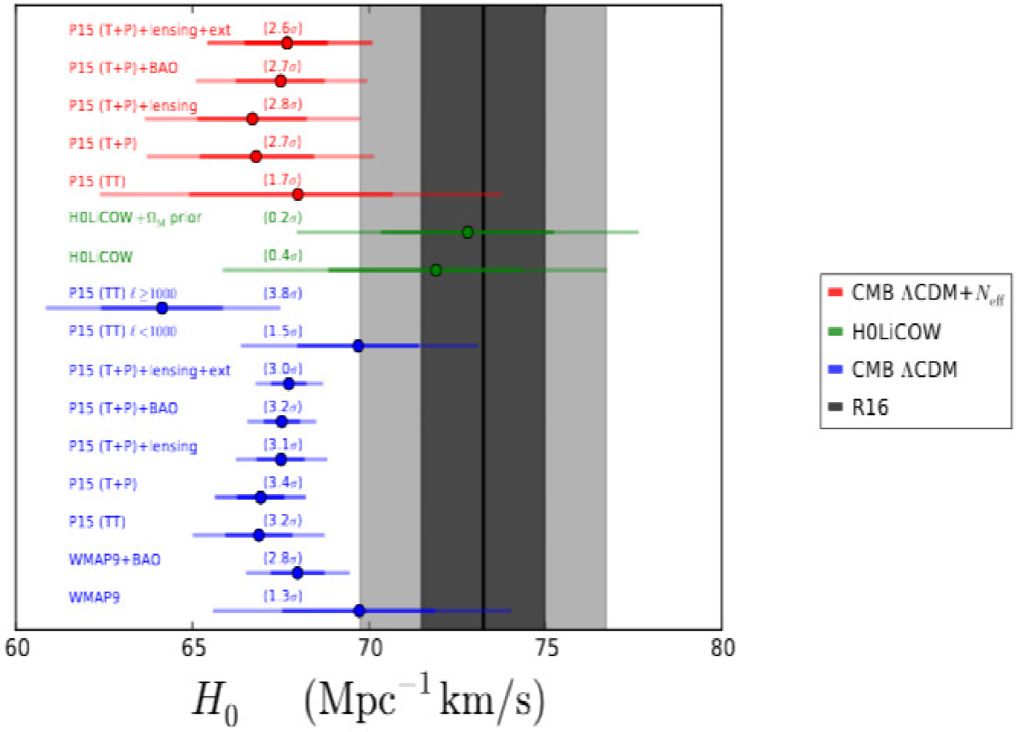
Parameter	Planck15/ΛCDM [12]	WMAP7/ΛCDM [45]
$\Omega_b h^2$	0.02225 ± 0.00016	0.02258 ± 0.00057
$\Omega_c h^2$	0.1198 ± 0.0015	0.1109 ± 0.0056
n_s	0.9645 ± 0.0049	0.963 ± 0.014
H_0	67.27 ± 0.66	71.0 ± 2.5
Ω_{0m}	0.3156 ± 0.0091	0.266 ± 0.025
w	-1	-1
σ_8	0.831 ± 0.013	0.801 ± 0.030



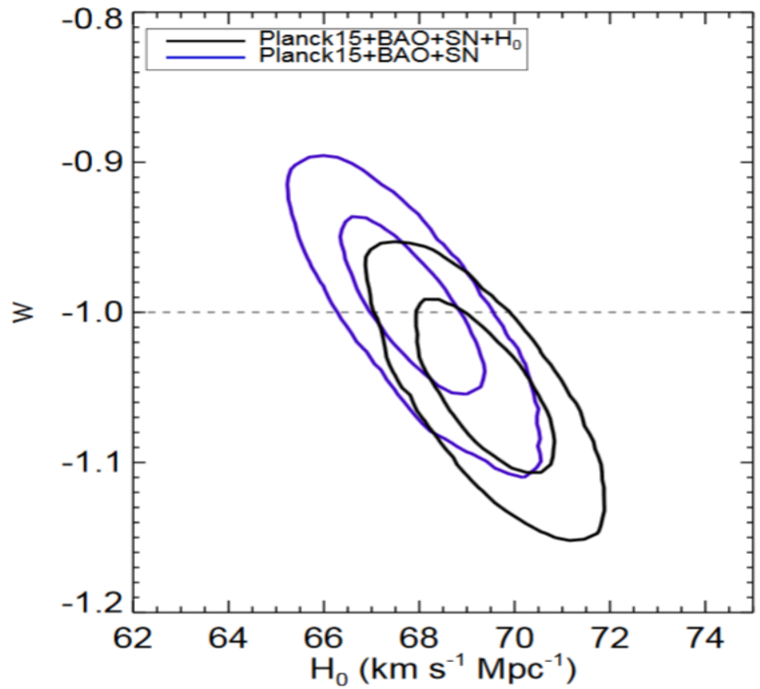
[Kazantzidis, Perivolaropoulos, PRD97]

Tension2 – H0

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of "gravitational power".



[Bernal, Verde, Riess, JCAP1610]



[Riess et al, Astrophys.J 826]