Exotic Compact Objects in Ricci-Based Gravity Theories

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In collaboration with

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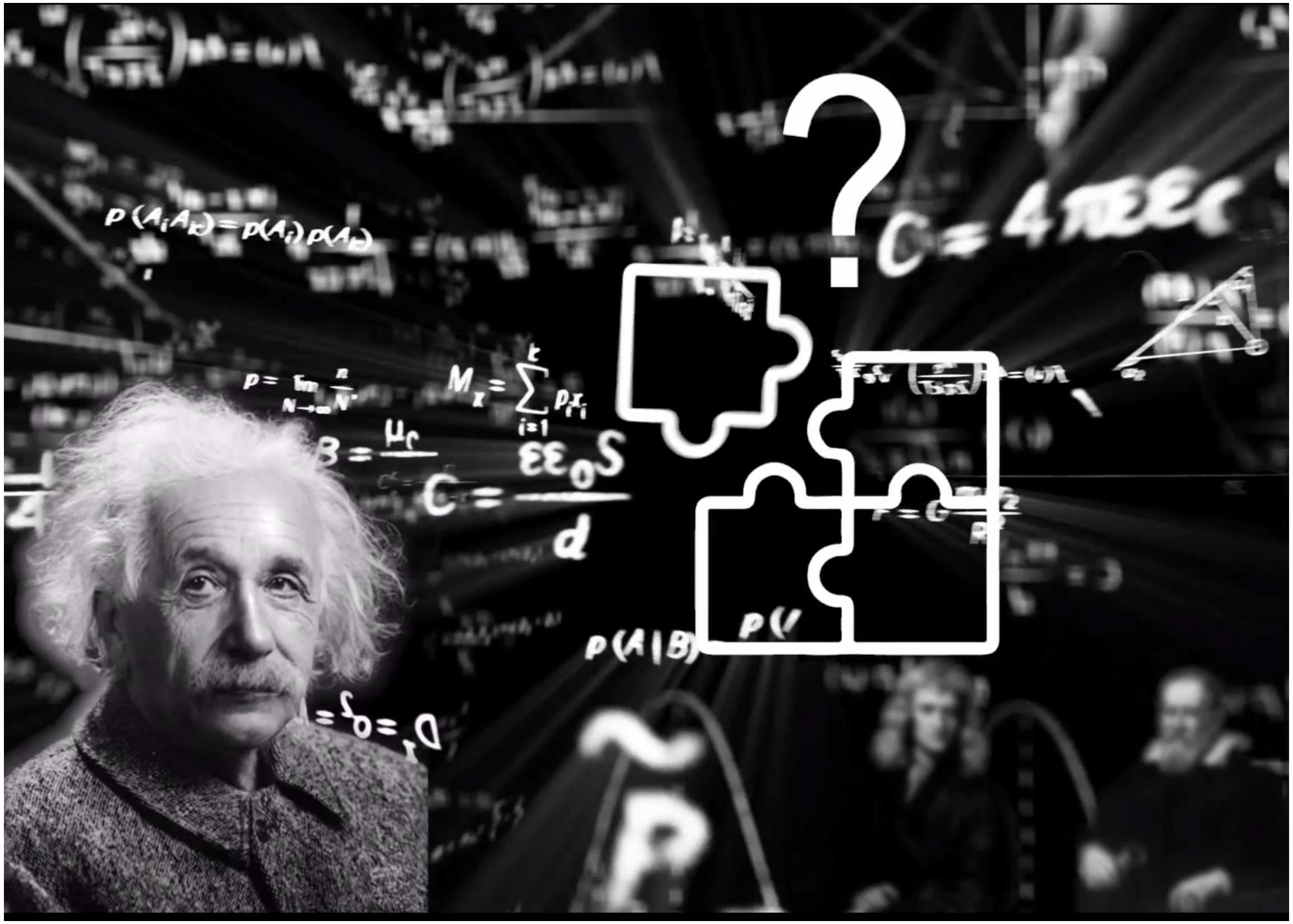
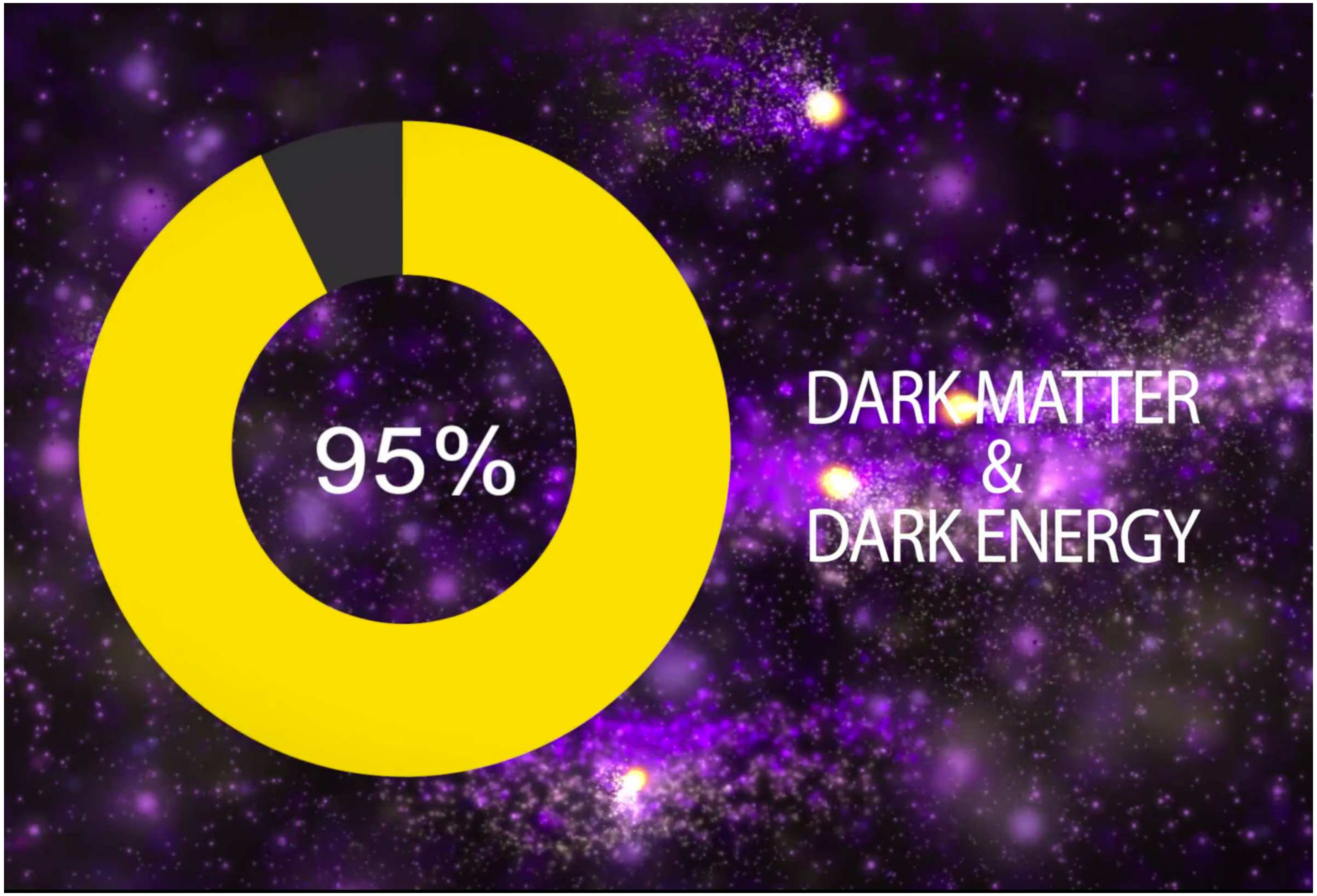
Motivations



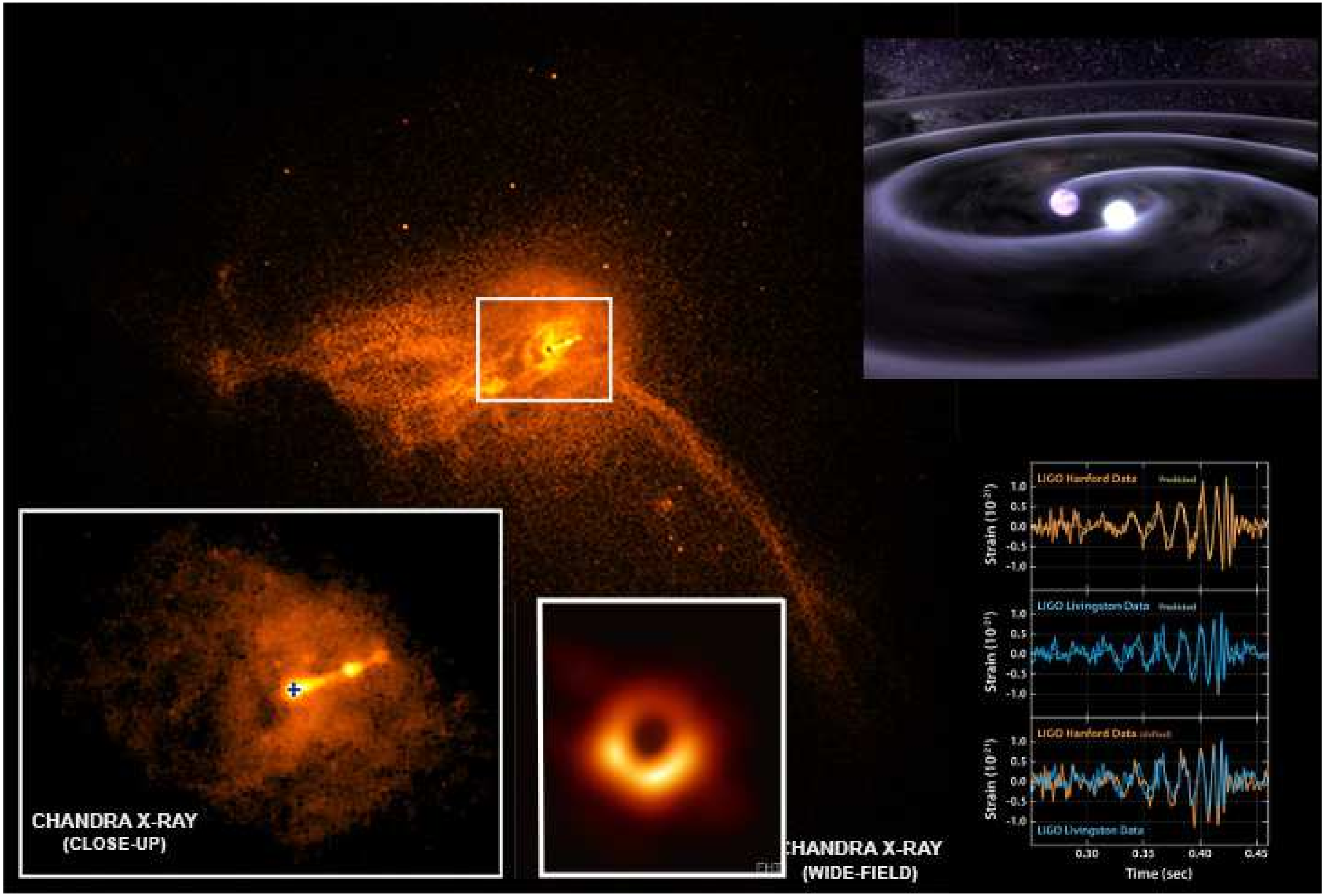
- Cosmological observations confirm many aspects of ΛCDM.
- But some pieces in the jigsaw do not fit well.
- Extensions of the matter sector seem natural.
- The gravitational side might also need some changes.

Motivations



- **CMB** finds $H_0 = 67$ while **local measurements** coincide in $H_0 = 73$.
- Dynamical dark energy or new gravitational dynamics at large scales (infrared)?
- What drives the cosmic expansion?

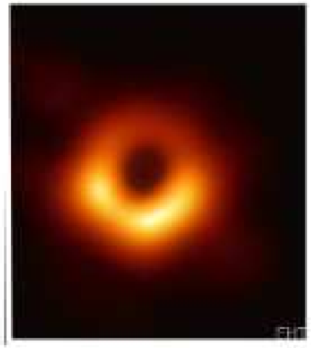
Motivations

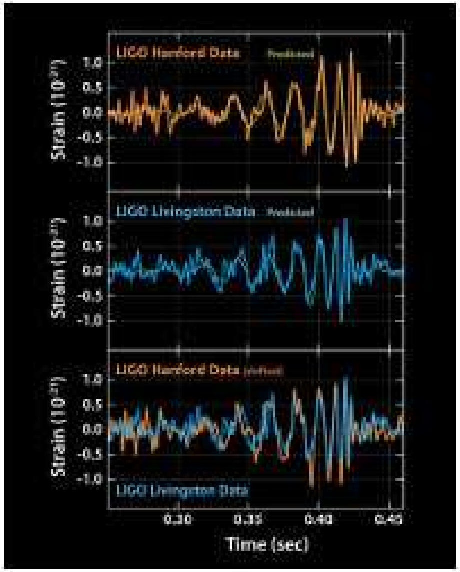


- Astrophysical properties of BHs have been verified.
- Conceptual problems remain: notion of singularity, Hawking radiation, ...

Motivations

- Gravitational wave astronomy and VLBI techniques pose new challenges:





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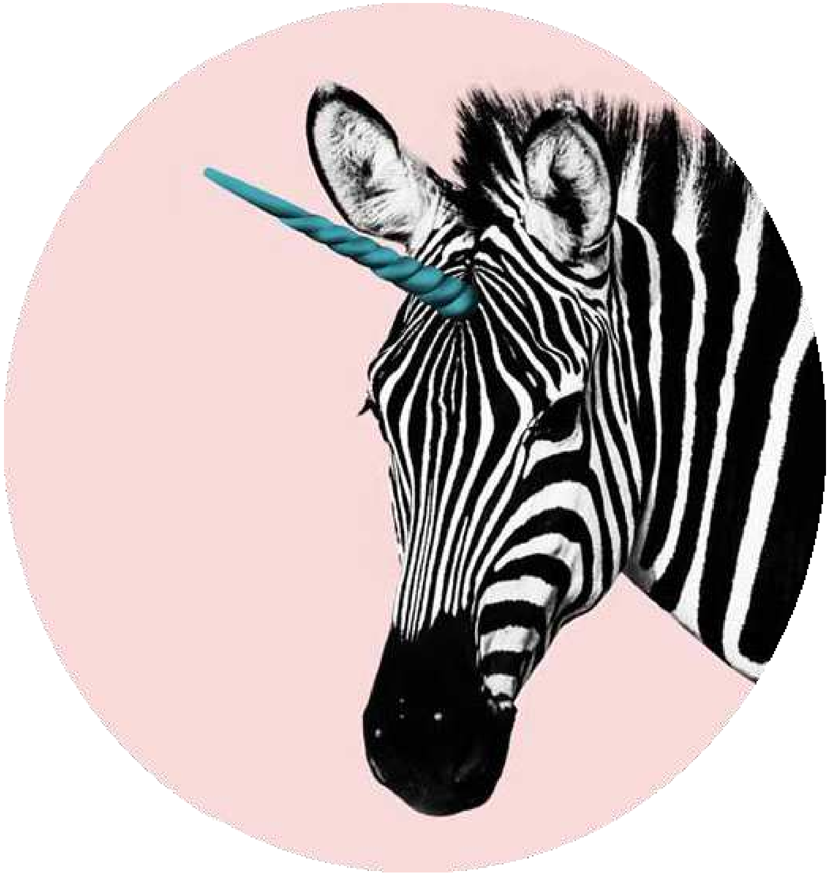
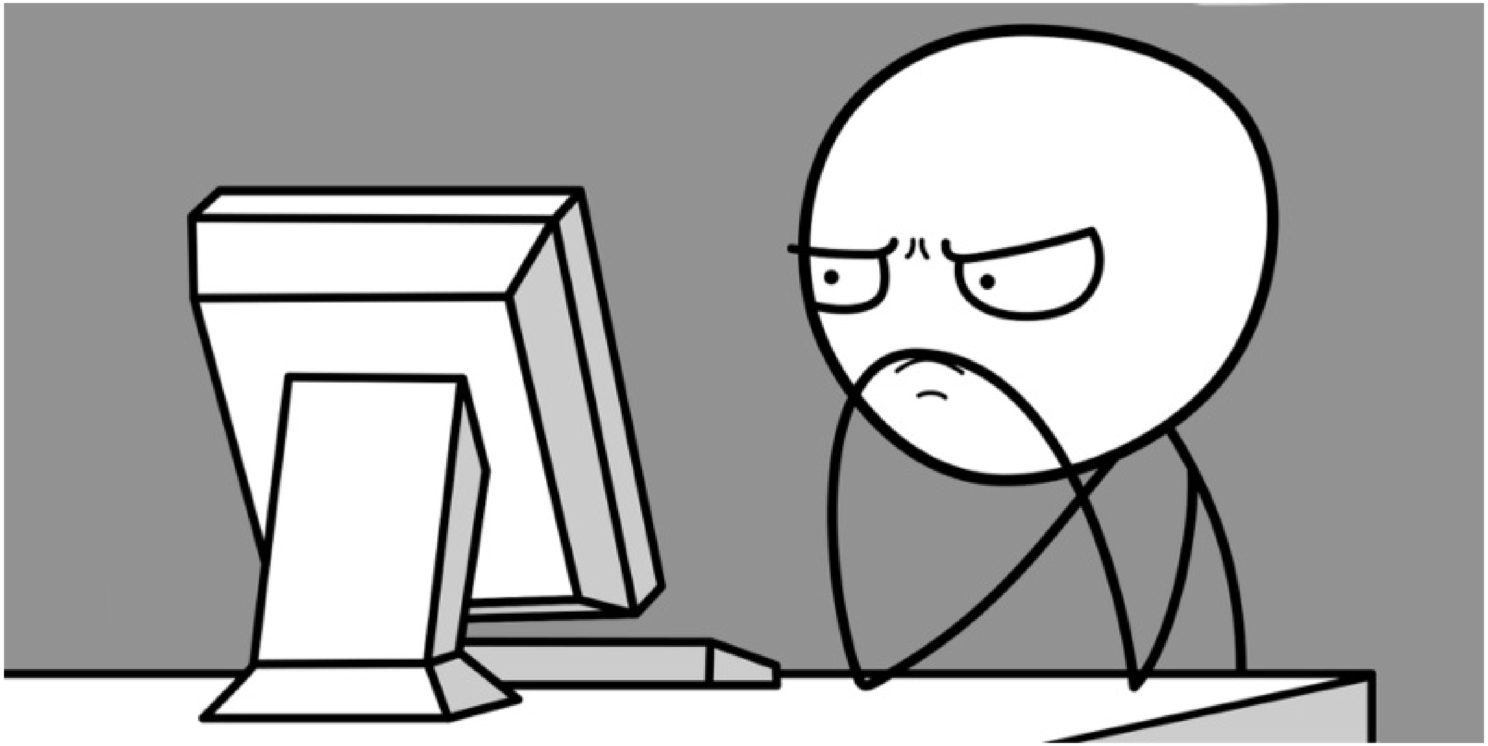


- Nature could bring surprises in unexpected ways!
- **New exact solutions are necessary to parametrize deviations from GR.**

Motivations

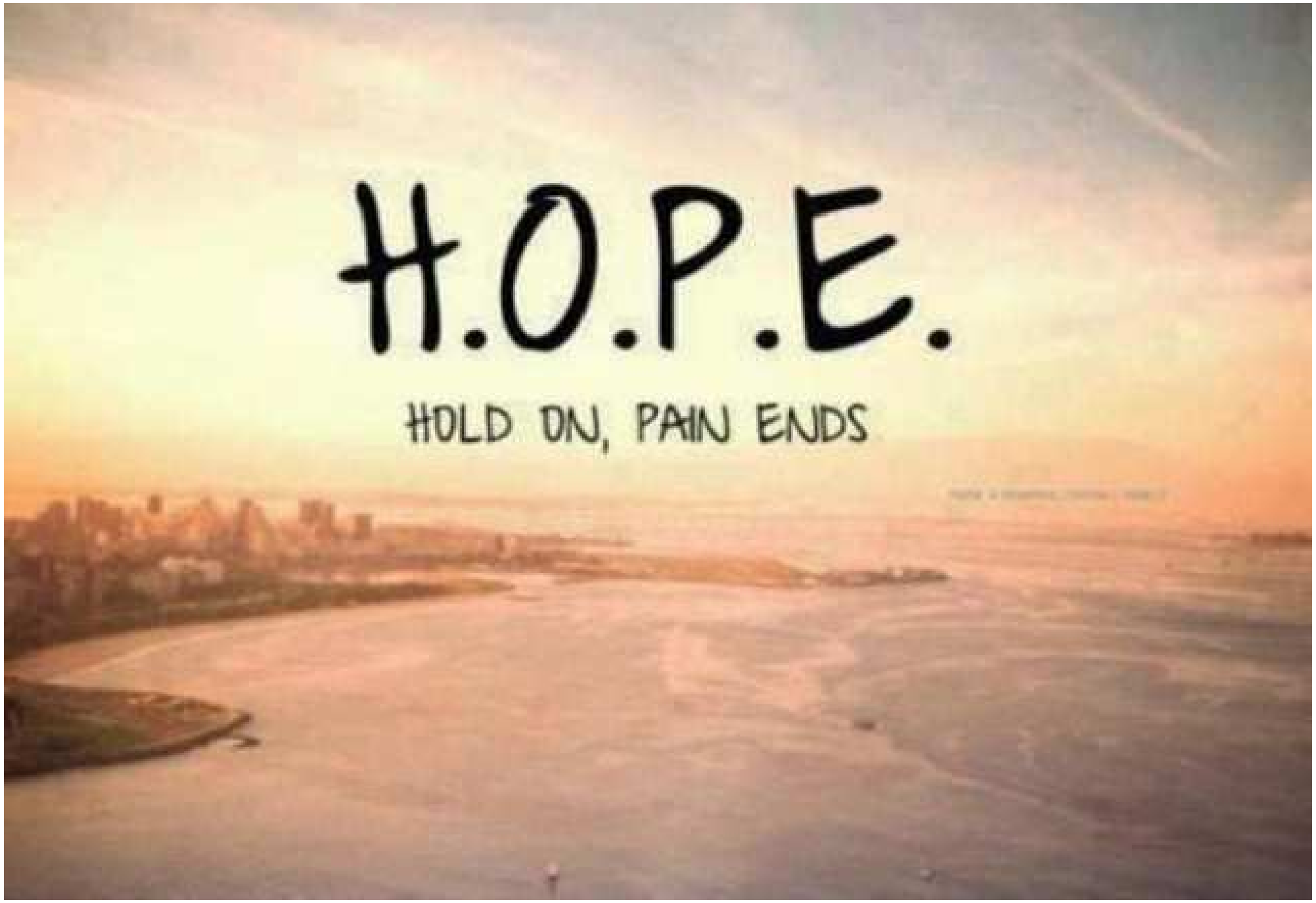
- Going beyond GR is a computational challenge:



- **We must build bridges** between modified gravity and GW astronomy/ numerical relativity.

Motivations

- There is hope for progress in certain families of modified theories of gravity:

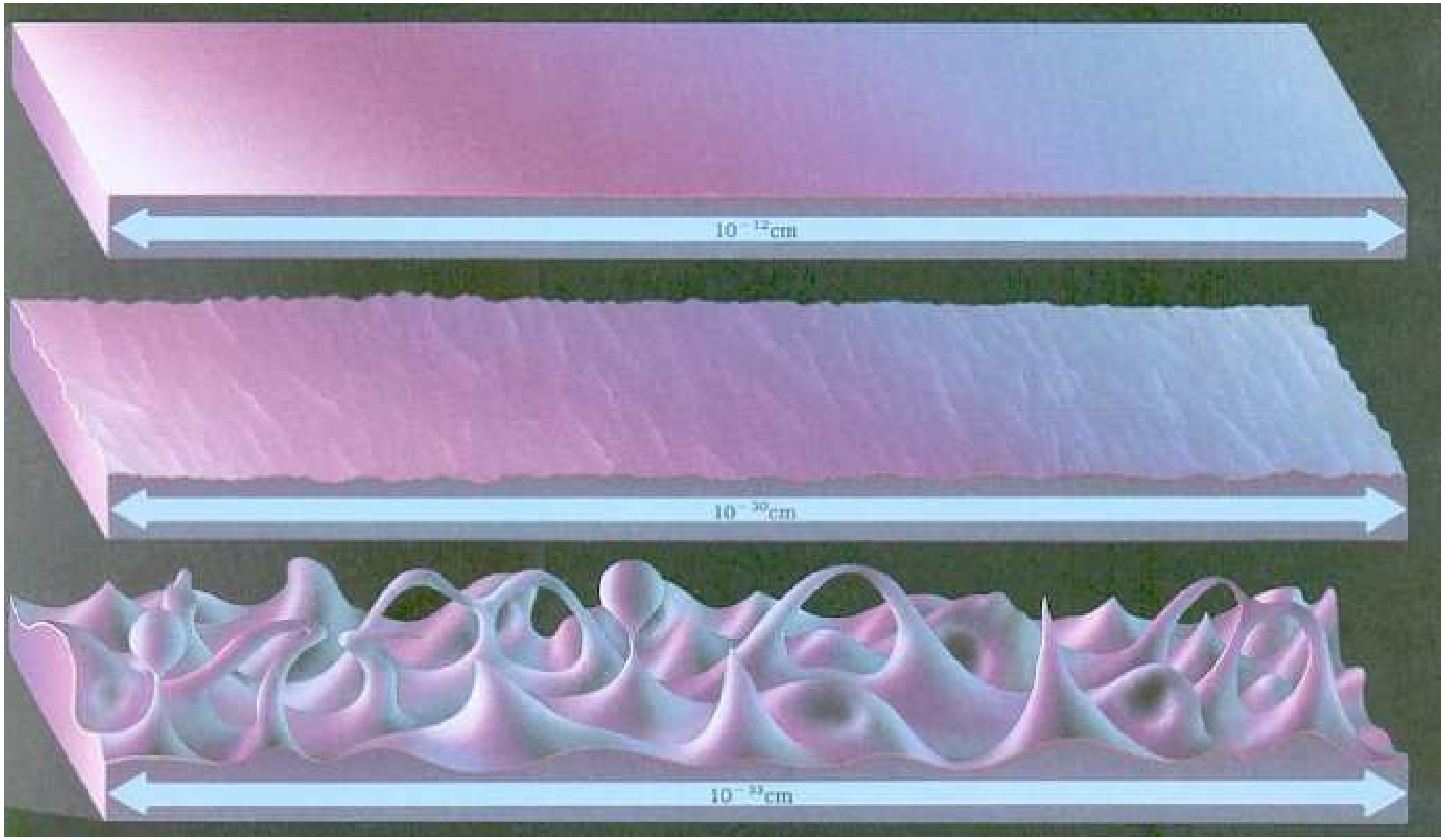


- Numerical and analytical methods can be **successfully implemented** in metric-affine gravity theories

Metric-Affine Modified Gravity

Space-time microstructure

- If topology change could occur dynamically:
  - The smoothness of Minkowski space disappears at Planckian scales.
  - Quantum fluctuations would lead to creation/annihilation of wormholes.
  - Fluxes through wormholes appear as pairs of elementary particles.



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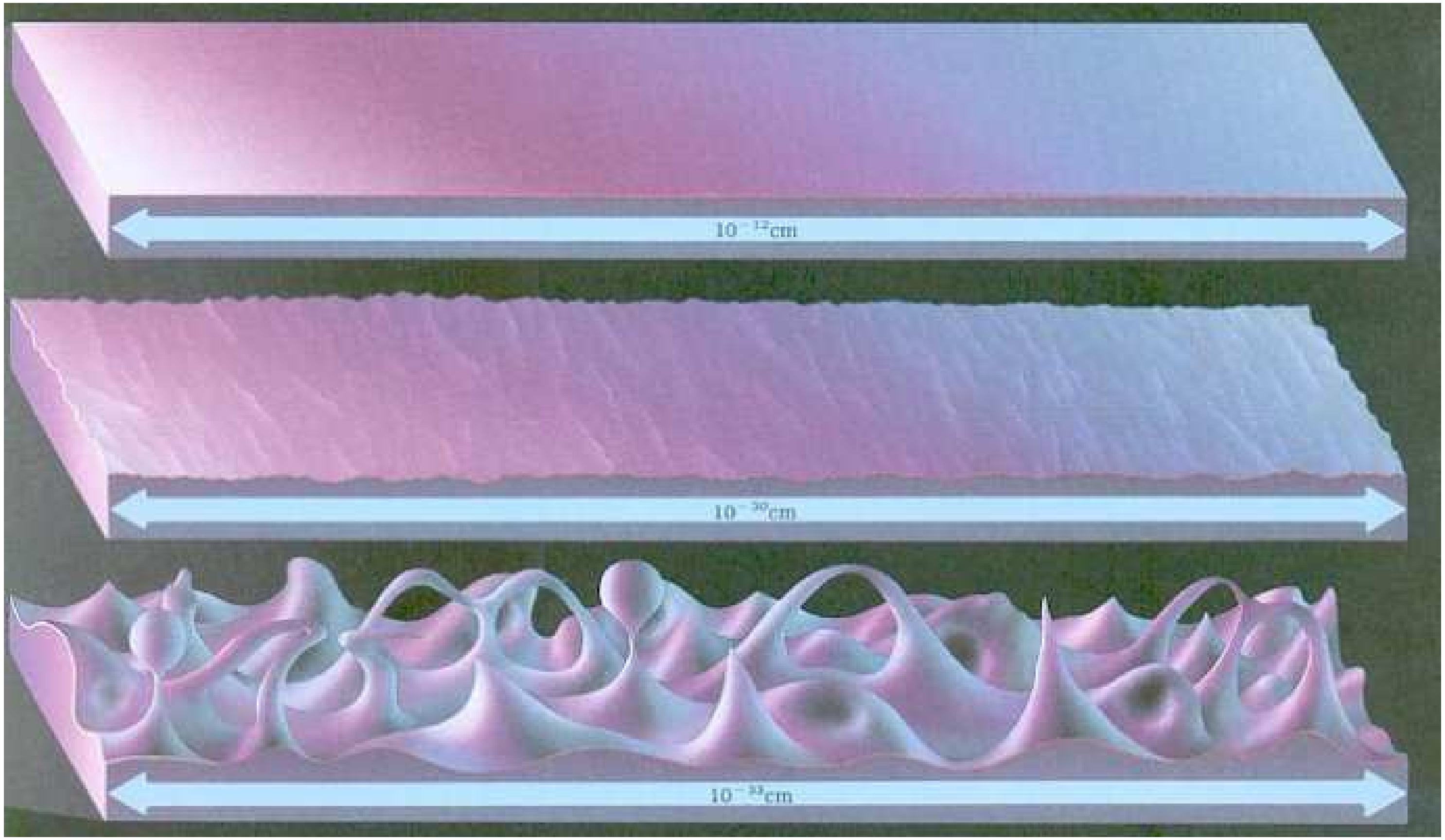
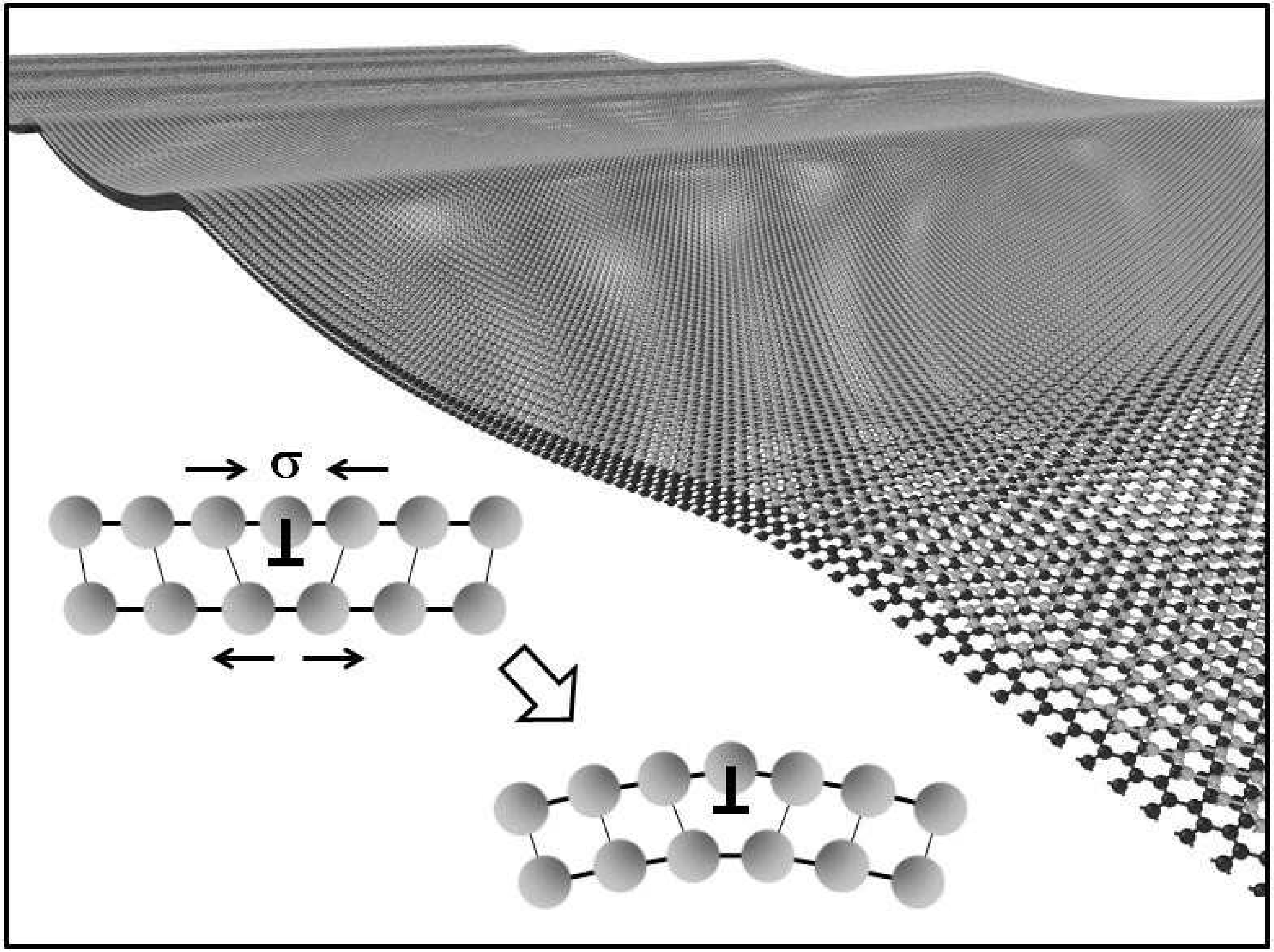


- What kind of framework should we use to describe this scenario?

Lessons from Condensed Matter Physics

- A microstructure with a macroscopic continuum limit is found in condensed matter systems such as **graphene** or Bravais crystals.



  - Wave propagation on the continuum effective geometry of bilayer graphene.

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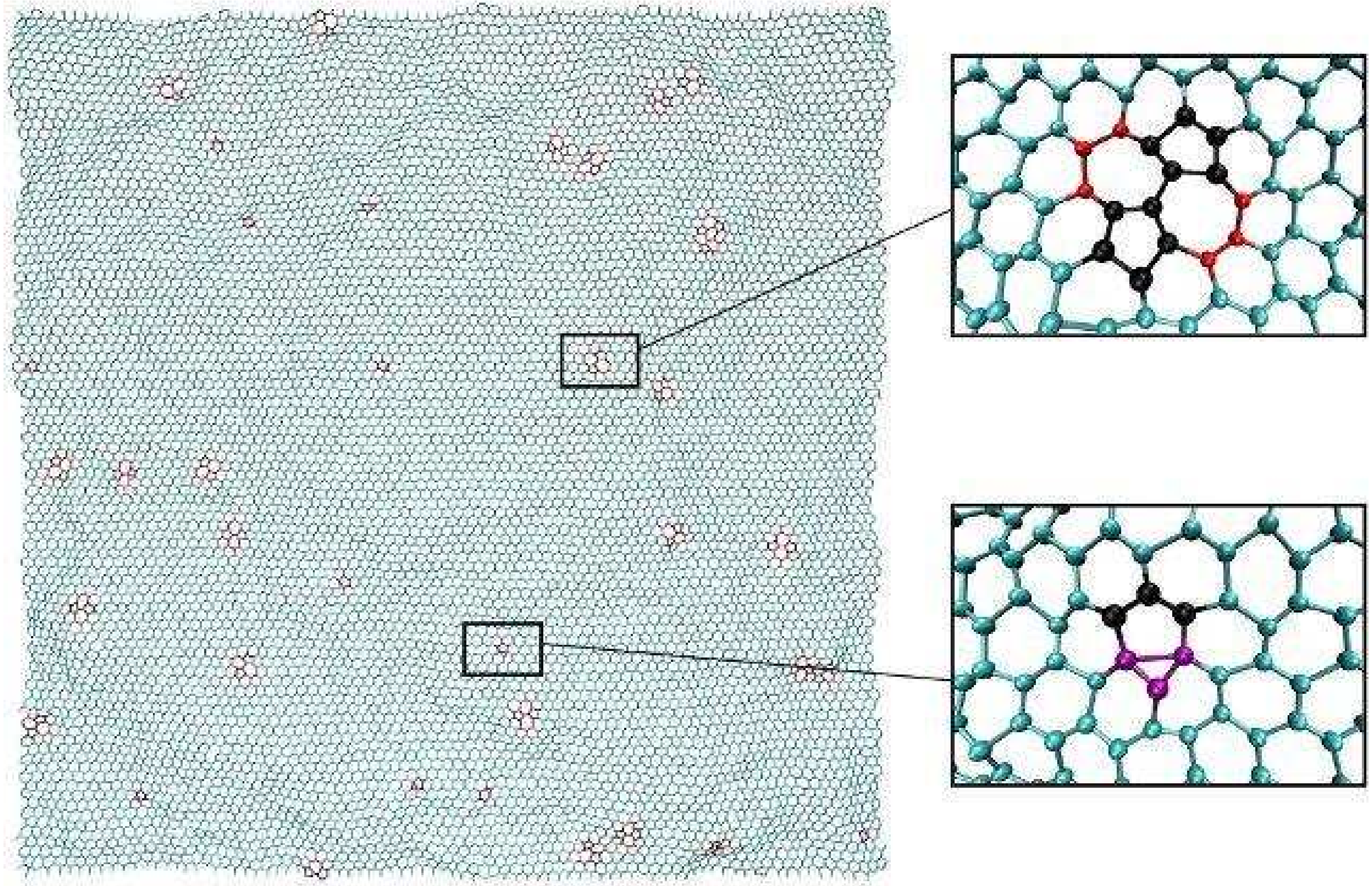
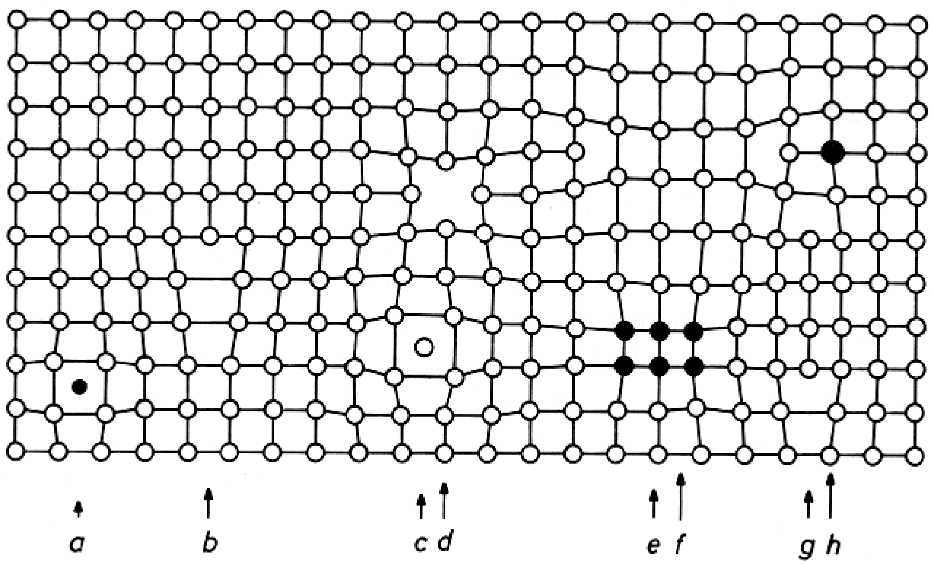


- Microscope image of a graphene layer with defects.

Metric-Affine geometries do exist in Nature

- A microstructure with a macroscopic continuum limit is found in condensed matter systems such as **graphene** or Bravais crystals.
- Crystalline structures may have different kinds of defects:



a) Interstitial impurity atom, b) Edge dislocation, c) Self interstitial atom
d) Vacancy, e) Precipitate of impurity atoms, f) Vacancy type dislocation loop,
g) Interstitial type dislocation loop, h) Substitutional impurity atom

- Point defects are related with non-metricity: $Q_{\alpha\mu\nu} = \nabla^{\Gamma}_{\alpha} g_{\mu\nu} \neq 0$.
- Dislocations (1D defects) generate torsion: $\Gamma^{\alpha}_{\mu\nu} \neq \Gamma^{\alpha}_{\nu\mu}$
- Metric-affine geometry could help better understand the transition from Quantum Gravity to **classical space-time**.

Metric-Affine - Vs - Metric theories

- We will be concerned with gravity theories in which metric and connection are a priori independent: $S=\int d^nx\sqrt{-g}L[g_{\mu\nu},\Gamma^\alpha_{\beta\gamma}]+S_m[g_{\mu\nu},\psi_m]$

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- Field equations in Palatini approach:

$$\delta S=\int d^n x\left[\sqrt{-g}\left(\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)\delta g^{\mu\nu}+\sqrt{-g}\,\frac{\delta L}{\delta \Gamma^{\alpha}_{\beta\gamma}}\delta\Gamma^{\alpha}_{\beta\gamma}\right]+\delta S_{matter}$$

$$\delta g^{\mu\nu} \Rightarrow \frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}=8\pi G T_{\mu\nu}$$

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(assuming no coupling of Γ to the matter)

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- Metric approach:

The relation $\delta\Gamma^\alpha_{\beta\gamma}=\frac{g^{\alpha\rho}}{2}\left[\nabla_\beta\delta g_{\rho\gamma}+\nabla_\gamma\delta g_{\rho\beta}-\nabla_\rho\delta g_{\beta\gamma}\right]$ implies

$\frac{\delta L}{\delta\Gamma^\alpha_{\beta\gamma}}\delta\Gamma^\alpha_{\beta\gamma}=\left\{g^{\alpha\mu}\frac{\delta L}{\delta\Gamma^\alpha_{\lambda\nu}}-\frac{g^{\alpha\lambda}}{2}\frac{\delta L}{\delta\Gamma^\alpha_{\mu\nu}}\right\}\nabla_\lambda\delta g_{\mu\nu}$ and leads to

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- The **Palatini** variation leads to second-order equations while the metric one induces higher-order derivatives. See also J. Beltrán and A. Delhom, 1901.08988.

Ricci-Based Gravity theories

Ricci-Based Gravity theories (RBGs)

- In GR replace $g^{\mu\nu}R_{\mu\nu}(\Gamma) \Rightarrow L_G[g^{\mu\alpha}R_{(\alpha\nu)}(\Gamma)]$
  - The connection can be solved as Levi-Civita of an auxiliary metric:
  $$\Gamma^{\mu}_{\alpha\beta} = \frac{h^{\mu\rho}}{2}\left(\partial_\alpha h_{\rho\beta} + \partial_\beta h_{\rho\alpha} - \partial_\rho h_{\alpha\beta}\right).$$
  - The two metrics are related by: $h_{\alpha\beta} = g_{\alpha\rho}\Omega^{\rho}{}_{\beta}$.
  - $\Omega^{\rho}{}_{\beta} = \Omega^{\rho}{}_{\beta}(T^{\mu}{}_{\nu})$ is a nonlinear function of the matter fields.
  - The metric field equations can be generically written as:
  $$G^{\mu}{}_{\nu}(h) = \frac{\kappa^2}{|\Omega|^{1/2}}\left[T^{\mu}{}_{\nu} - \delta^{\mu}_{\nu}\left(L_G + \frac{T}{2}\right)\right].$$

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- These theories generically recover GR+Λ in vacuum:
  - Only two propagating d.o.f. which travel at the speed of light.
  - Weak-field limit satisfied (unless anomalous behavior at low curvatures).
  - $h_{\mu\nu}$ is sensitive to the total energy content.
  - $g_{\mu\nu}$ also feels the local energy-densities $\Rightarrow$ $\nabla^{\Gamma}_{\alpha}g_{\mu\nu} \neq 0$.
  - $Q_{\alpha\mu\nu} = \nabla^{\Gamma}_{\alpha}g_{\mu\nu} \neq 0$ generated by stress-energy densities $\leftrightarrow$ crystal deffects.

Relating RBGs with GR

- The field equations of RBGs can be put into correspondence with those of GR:

$$L_{RBG}+L_m(g_{\mu\nu},\psi) \quad \Leftrightarrow \quad R+\tilde{L}_m(q_{\mu\nu},\psi)$$

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- This correspondence has been worked out for several gravity+matter models:
  - Scalar fields: **arXiv:1801.10406 [gr-qc]** and **arXiv:1810.04239 [gr-qc]** .
  - Anisotropic fluids / electric fields: **arXiv:1807.06385 [gr-qc]**.
  - Electromagnetic fields: **arXiv:1907.04183 [gr-qc]**

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  - Electromagnetic fields: **arXiv:1907.04183 [gr-qc]**
- Not something you can do on the back of an envelope!

Example: $f(R)$ theories

- Consider $f(R) = R - \sigma R^2$ coupled to $L_m(X) = X$, with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
  - The map to GR leads to $\tilde{L}_m(Z) = Z + \sigma\kappa^2 Z^2$, with $Z = q^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
  - Thus quadratic $f(R)$ plus free scalar leads to GR plus quadratic scalar.

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  - Thus quadratic $f(R)$ plus free scalar leads to GR plus quadratic scalar.
- Consider the map from GR with $K(Z,\phi) = Z - 2V(\phi)$ to $f(R) = R - \sigma R^2$
  - The map leads to $P(X,\phi) = \frac{X - \sigma\kappa^2 X^2}{1 - 8\sigma\kappa^2 V(\phi)} - \frac{2V(\phi)}{1 - 8\sigma\kappa^2 V(\phi)}$
  - In the purely kinetic case: $P(X) = X - \sigma\kappa^2 X^2$
  - Thus GR plus free scalar leads to quadratic $f(R)$ plus quadratic scalar.
  - Transforming back this $P(X,\phi)$ model to GR one recovers $Z - 2V(\phi)$.

Example: Born-Infeld gravity+fluids

- Let us focus on $S_{EiBI} = \frac{1}{\kappa^2\epsilon}\int d^4x\left[\sqrt{|g_{\mu\nu}+\epsilon R_{(\mu\nu)}|}-\lambda\sqrt{-g}\right]$ with a fluid
  - In this theory $|\hat{\Omega}|^{\frac{1}{2}}[\Omega^{-1}]^{\mu}{}_{\nu} = \lambda\delta^{\mu}_{\nu} - \epsilon\kappa^2 T^{\mu}{}_{\nu}$ and $\mathcal{L}_{EiBI} = (|\hat{\Omega}|^{\frac{1}{2}} - \lambda)/\epsilon\kappa^2$

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- Using the Einstein-frame variables, $\tilde{T}^{\mu}{}_{\nu}$, the deformation matrix becomes:

$$[\Omega^{-1}]^{\mu}{}_{\nu} = \left(1 - \frac{\epsilon\kappa^2}{2}[\tilde{\rho} - \tilde{p}_r]\right)\delta^{\mu}{}_{\nu} - \epsilon\kappa^2(\tilde{\rho} + \tilde{p}_{\perp})v^{\mu}v_{\nu} - \epsilon\kappa^2(\tilde{p}_r - \tilde{p}_{\perp})\xi^{\mu}\xi_{\nu}$$

- Given that $g_{\mu\nu} = h_{\mu\alpha}[\Omega^{-1}]^{\alpha}{}_{\nu}$, we find:

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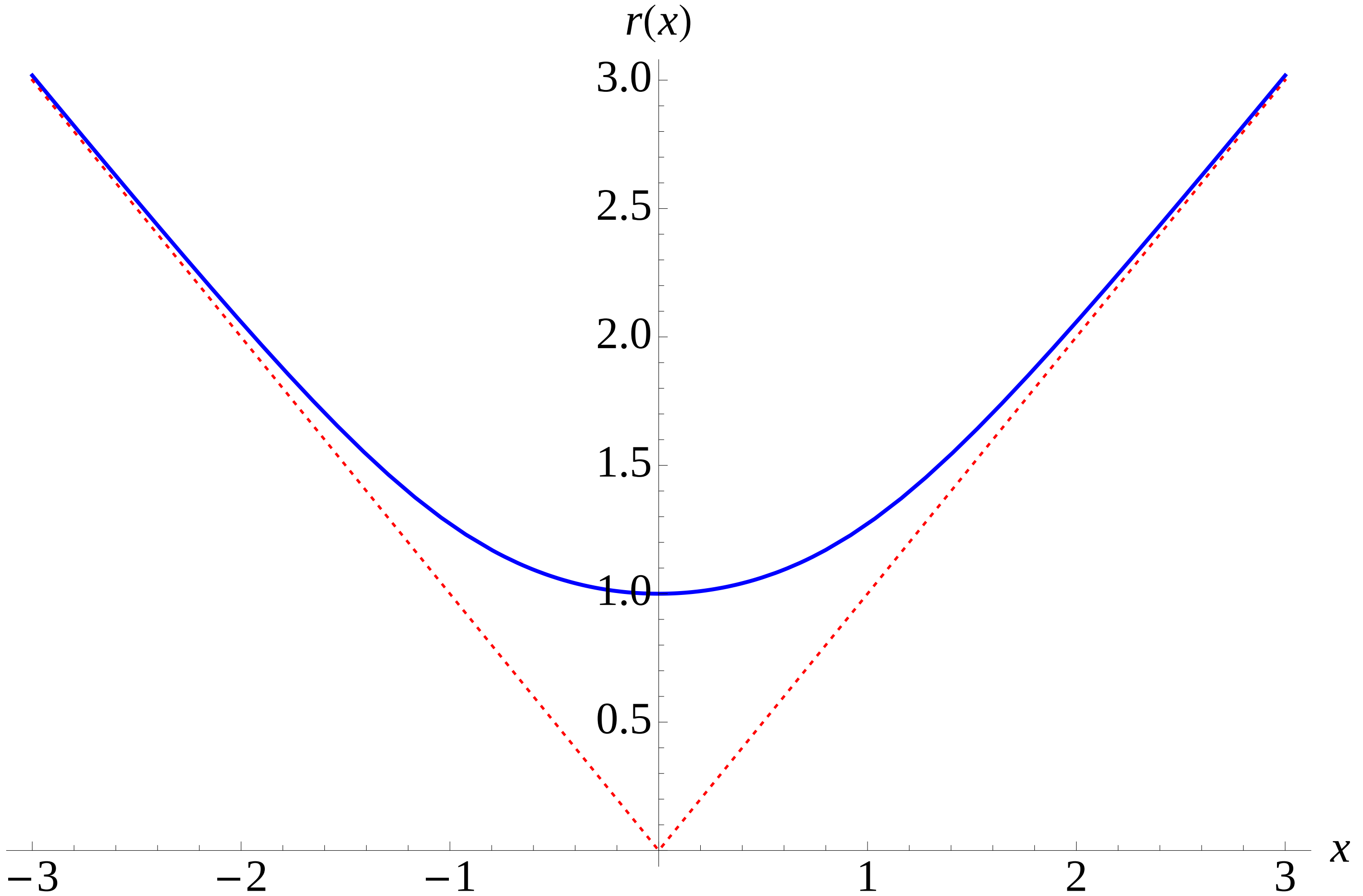
> Once a solution for an anisotropic fluid is known in GR, this generates new solutions in the Born-Infeld gravity theory.

- For a Maxwell field in GR, the mapping to Born-Infeld gravity leads to a nonlinear electrodynamics theory of the form $\varphi(X) = \frac{1}{2\epsilon}\left(1-\sqrt{1-4\epsilon X}\right)$, which is exactly of the Born-Infeld type!!!

Charged BHs have wormhole structure

- In Born-Infeld gravity + spherical Maxwell electric field, the radial function behaves as:



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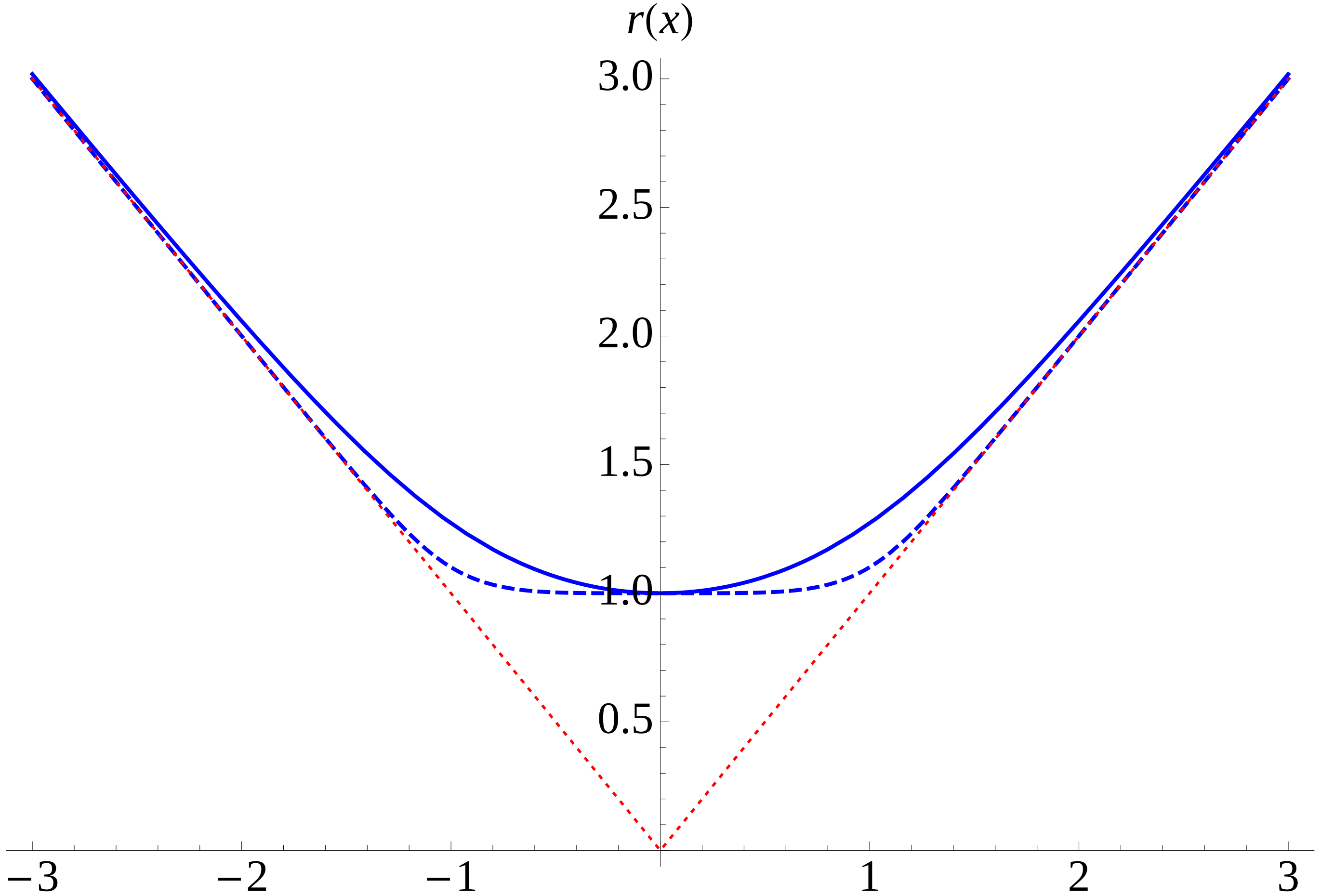

- $ds^2 = -A(x)dt^2 + \frac{1}{B(x)}dx^2 + r^2(x)d\Omega^2_{D-2}$. $D = 4$ (solid), $D = 7$ (dashed)
- In higher dimensions the transient is more pronounced

Curvature scalars and BH remnants

- Depending on $\delta_1 = \frac{r_q^2}{2r_S r_c}$, with $r_c^2 = r_q l_\varepsilon$ and $\delta_c = 0.572$ we have
  - If $\delta_1 > \delta_c$ $\Rightarrow$ Reissner-Nordstrom like solutions.
  - If $\delta_1 < \delta_c$ $\Rightarrow$ Schwarzschild like solutions.
  - If $\delta_1 = \delta_c$ $\Rightarrow$ Regular solutions:
    - If $N_q > 16 \Rightarrow$ Schwarzschild like (with black bounce).
    - If $N_q < 16 \Rightarrow$ two Minkowskian spaces joined by a traversable wormhole.

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- Curvature scalars:

$$R(g) \approx \left(-4 + \frac{16\delta_c}{3\delta_2}\right) - \frac{1}{2\delta_2}\left(1 - \frac{\delta_c}{\delta_1}\right)\left[\frac{1}{(z-1)^{3/2}} - O\left(\frac{1}{\sqrt{z-1}}\right)\right]$$

$$R_{\mu\nu}R^{\mu\nu} \approx \left(10 + \frac{86\delta_1^2}{9\delta_2^2} - \frac{52\delta_1}{3\delta_2}\right) + \left(1 - \frac{\delta_c}{\delta_1}\right)\left[\frac{6\delta_2 - 5\delta_1}{3\delta_2^2 (z-1)^{3/2}} + \dots\right] + \left(1 - \frac{\delta_c}{\delta_1}\right)^2 \left[\frac{1}{8\delta_2^2 (z-1)^3} - \dots\right]$$

$$(R^\alpha{}_{\beta\mu\nu})^2 \approx \left(16 + \frac{88\delta_1^2}{9\delta_2^2} - \frac{64\delta_1}{3\delta_2}\right) + \left(1 - \frac{\delta_c}{\delta_1}\right)\left[\frac{2(2\delta_1 - 3\delta_2)}{3\delta_2^2 (z-1)^{3/2}} + \dots\right] + \left(1 - \frac{\delta_c}{\delta_1}\right)^2 \left[\frac{1}{4\delta_2^2 (z-1)^3} + \dots\right]$$

Geodesic completeness in Born-Infeld

- The equation that governs the evolution of geodesics in this space-time is:

  $$\frac{1}{\sigma_+^2}\left(\frac{dx}{d\lambda}\right)^2 = E^2 - V_{eff} \quad , \text{ with } V_{eff} \equiv \left(\kappa + \frac{L^2}{r^2}\right) A(r) \ .$$

  - Where $\kappa = 0$ for null geodesics and $\kappa = 1$ for time-like geodesics.
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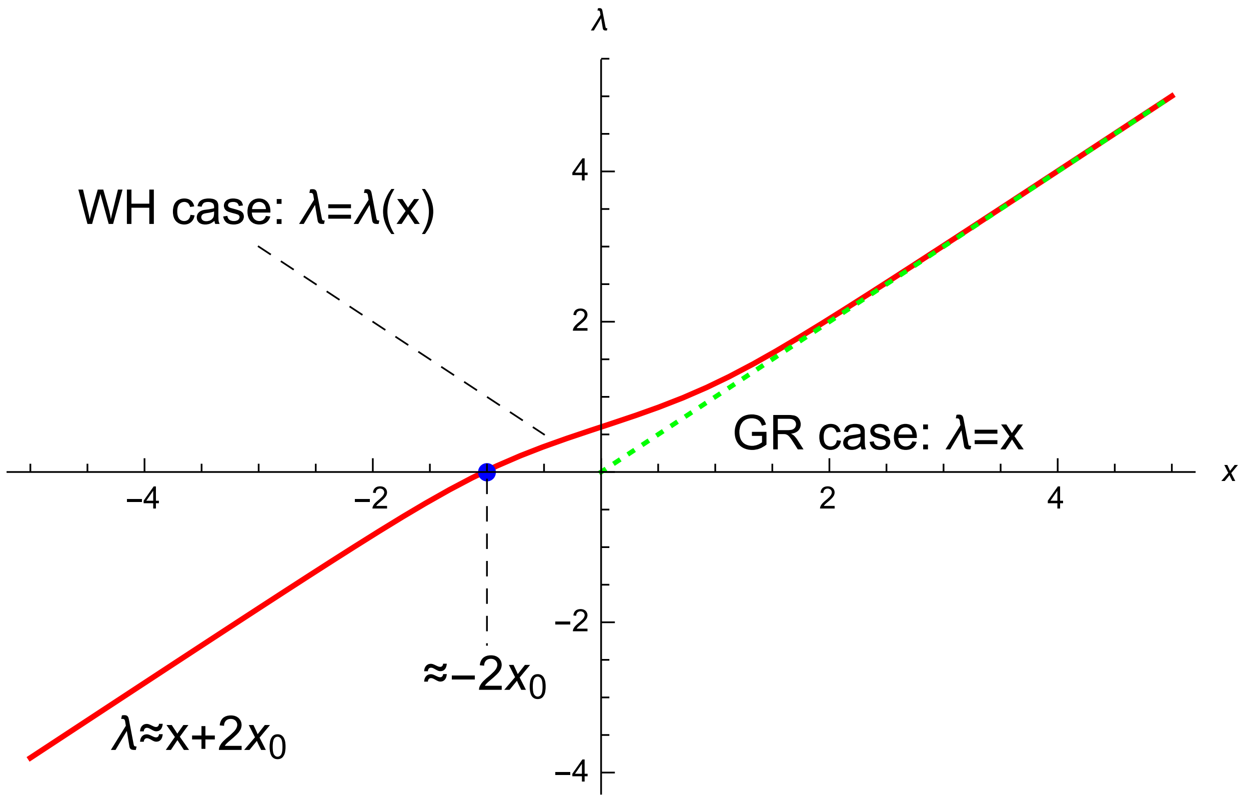
  - Where $\kappa = 0$ for null geodesics and $\kappa = 1$ for time-like geodesics.
  - L^2 and E^2 are the angular momentum and energy per unit mass.
- For null radial geodesics $V_{eff} = 0$



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- But they should neither be created nor destroyed.

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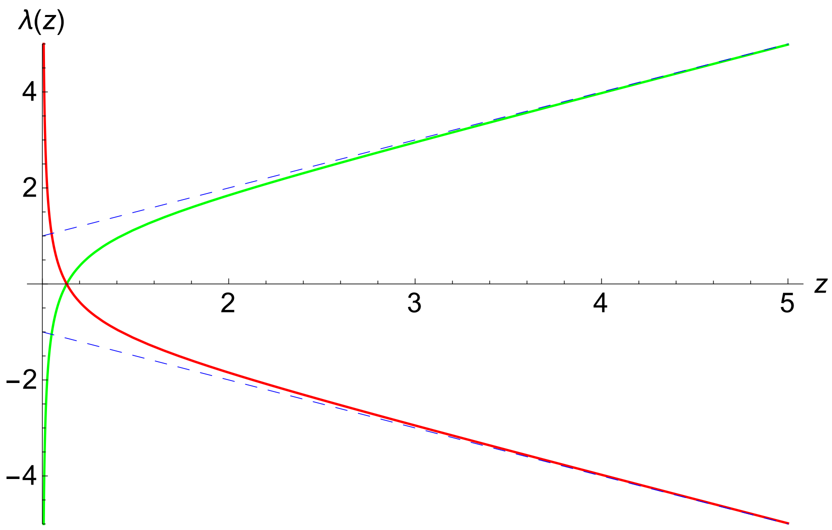
- Observers can suffer deformations and tidal forces.
- But they should neither be created nor destroyed.
- Existence is more important than suffering!

NEDs in $f(R) = R - \sigma R^2$



- As $z \to \infty$: $\pm E\lambda(z) \approx z \approx x$.
- As $z \to 1$: $\pm E\lambda(z) \approx -\frac{1}{\sqrt{2\alpha+2\sqrt{z-1}}} \approx -\frac{1}{|\tilde{x}|}$.
- Geodesically complete space.

NEDs in $f(R) = R - \sigma R^2$

$$f(R) = R - \sigma R^2 \qquad \rho = \frac{\rho_m}{z^4 - s_\beta}$$

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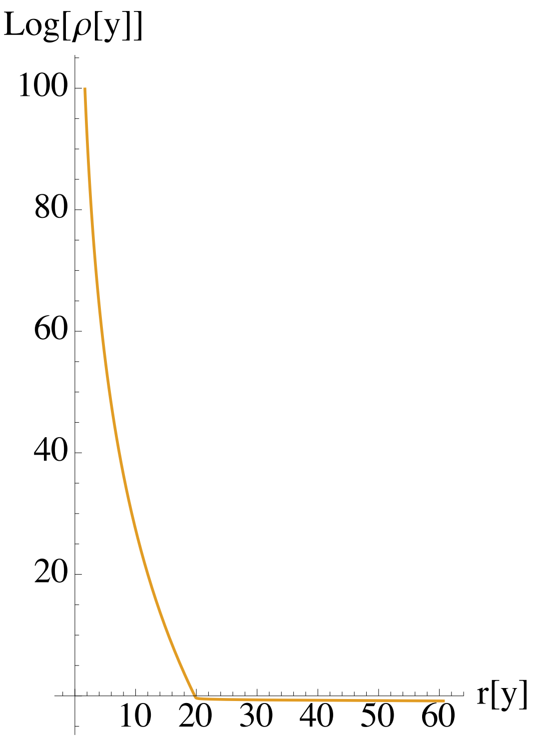
	Wormhole	Metric components	Energy density	Curvature scalars	Geodesics
Case I ($\sigma > 0, \beta < 0$)	YES if $\gamma > 1$ NO if $\gamma < 1$	Divergent	Finite	Divergent	Complete if $\gamma > 1$ Incomplete if $\gamma < 1$
Case II ($\sigma > 0, \beta > 0$)	NO	Finite	Divergent	Finite	Incomplete
Case III ($\sigma < 0, \beta < 0$)	NO	Divergent if $\delta_1 \neq \delta_c^{(\gamma)}$ Finite if $\delta_1 = \delta_c^{(\gamma)}$	Finite	Divergent if $\delta_1 \neq \delta_c^{(\gamma)}$ Finite if $\delta_1 = \delta_c^{(\gamma)}$	Incomplete? if $\delta_1 > \delta_c^{(\gamma)}$ Complete if $\delta_1 = \delta_c^{(\gamma)}$ (de Sitter core) Incomplete if $\delta_1 < \delta_c^{(\gamma)}$
Case IV ($\sigma < 0, \beta > 0$)	YES	Divergent	Finite	Divergent	Complete

Table I. Summary of the features of the four families of configurations studied in Sec. V (the case $\{\sigma > 0, \beta < 0\}$ with $\gamma = 1$ hides some peculiarities, see Sec. V A, so it is not contained in this table). The metric components, energy density and curvature scalars refers to the behaviour at the wormhole throat (when it exists) or otherwise at the innermost region of the solutions. Incomplete geodesics refer to the existence of (at least) a single incomplete null or timelike geodesic curve. The breakdown of the correlations among these three concepts is clear.

In GR geodesic incompleteness occurs simultaneously with curvature and matter divergences. This degeneracy is broken beyond GR.

Scalar compact object in GR

- Wyman (PRD24,1981) obtained an exact, asymptotically flat, free scalar field solution:
  - $ds^2_{GR} = -e^{\nu}dt^2 + \frac{e^{\nu}}{W^4}dy^2 + \frac{1}{W^2}(d\theta^2 + \sin\theta^2 d\varphi^2)$
  - Here $\Box\phi = 0$ becomes $\phi_{yy} = 0 \Rightarrow \phi = y$.
  - $e^{\nu} = e^{\beta y}$, $W = e^{\beta y/2}\sinh(\gamma y)/\gamma$,
  - $\gamma \equiv \sqrt{\beta^2 + 2\kappa^2}/2$ and $\beta = -2M$.



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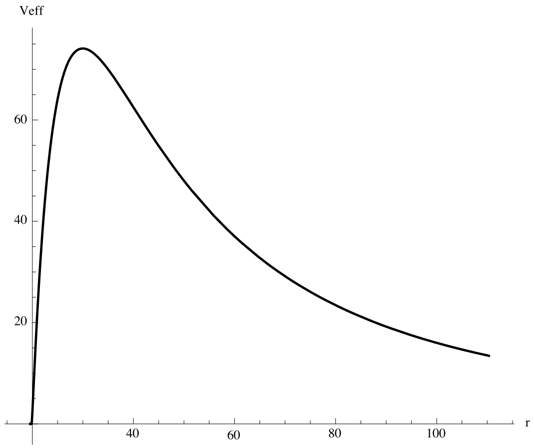

- Properties of this solution:
  - The far limit is $y \to 0$, where $W \approx y$.
  - Energy density concentrated inside the Scharzschild radius.
  - No event horizon but an ISCO at $\sim 3M$.
  - Geodesics reach the center in finite affine time.
  - Strong curvature divergence at the center.

Mapping the scalar field into $f(R)$

- The new solution looks like:
  - $ds^2_{f(R)} = \frac{1}{f_R(Z)} ds^2_{GR}$
  - Here $Z = W^4 e^{-\nu}$ and $f_R = 1/(1 + 2\sigma\kappa^2 Z)$.
  - Two families of solutions depending on σ.

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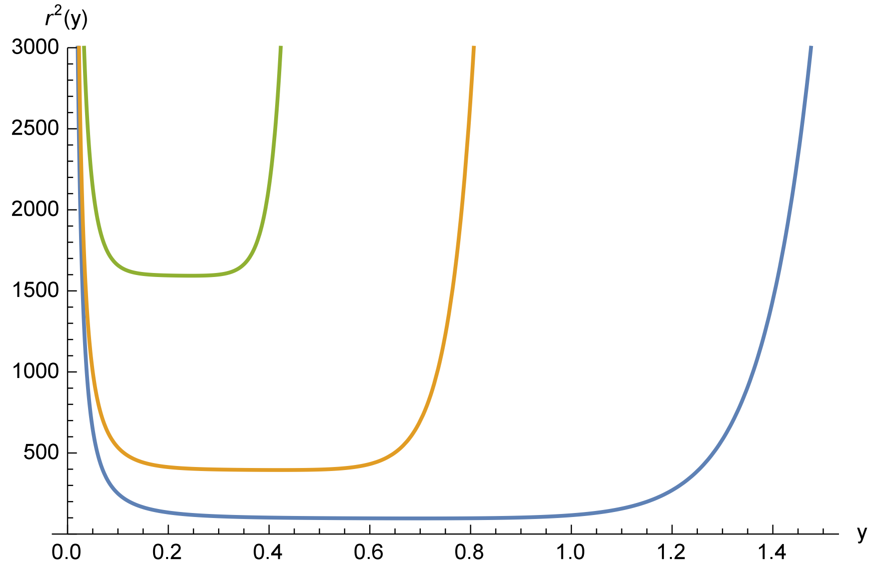

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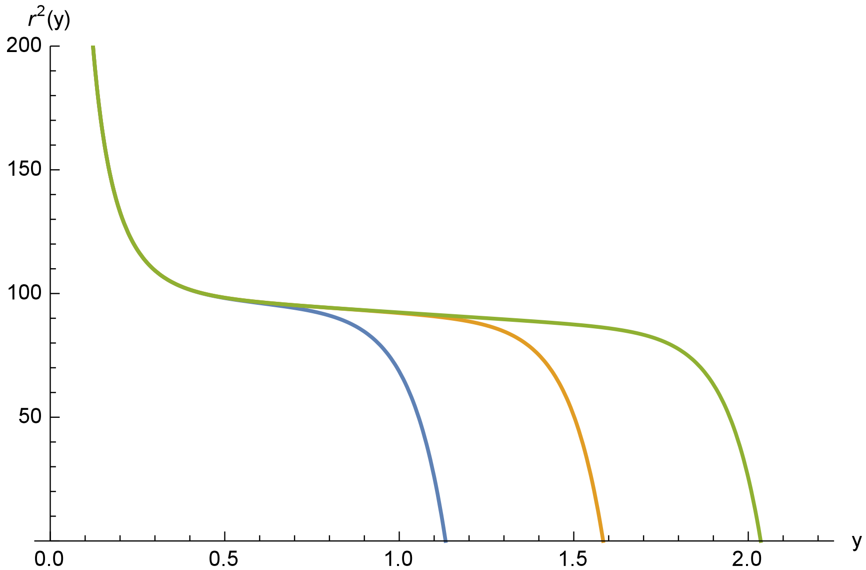
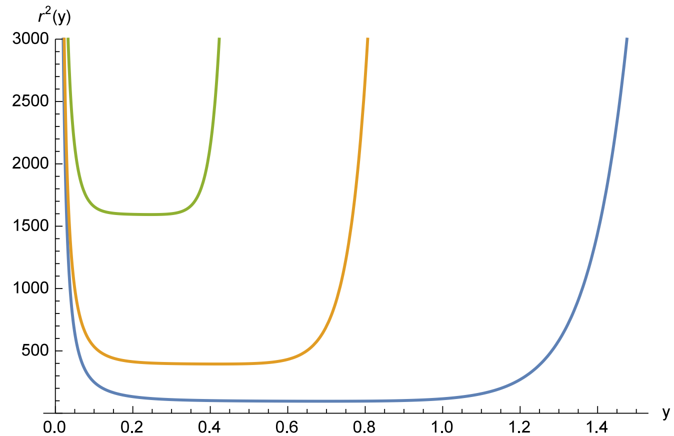


$\sigma < 0$ yields a wormhole.



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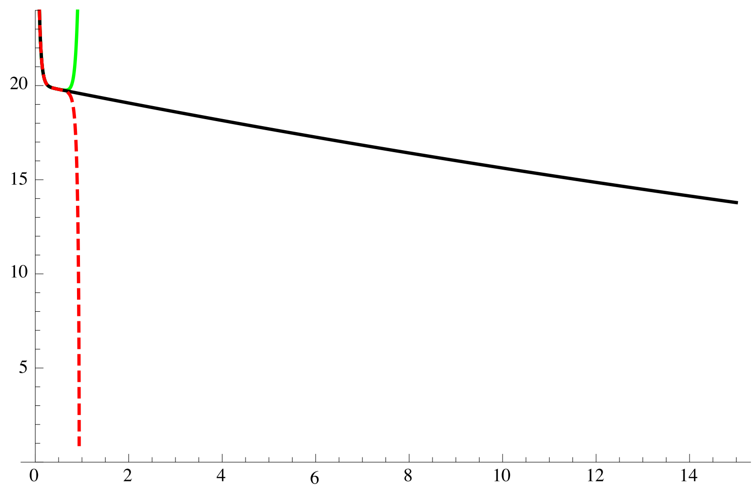
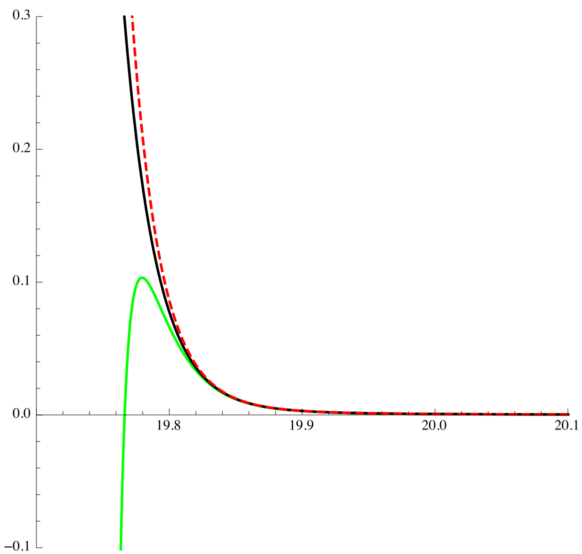
Radial Functions



Energy density



Mapping the scalar field into EiBI

- The solution looks pretty much like the GR one:

$$ds^2_{GR} = -e^{\nu} dt^2 + \left(\frac{e^{\nu}}{W^4} - \varepsilon\kappa^2\right) dy^2 + \frac{1}{W^2}(d\theta^2 + \sin\theta^2 d\varphi^2)$$

  - GR is recovered when $\varepsilon \to 0$.
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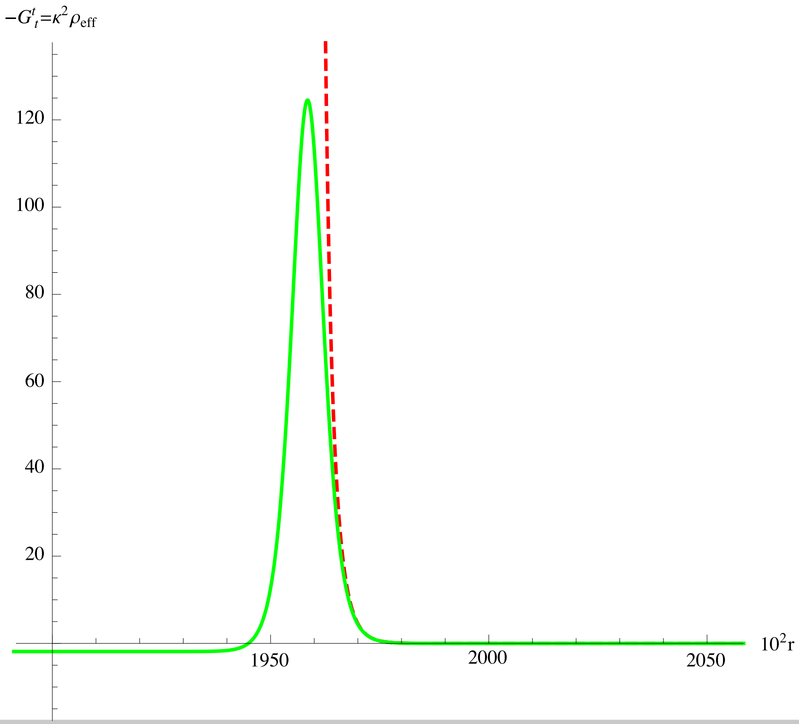
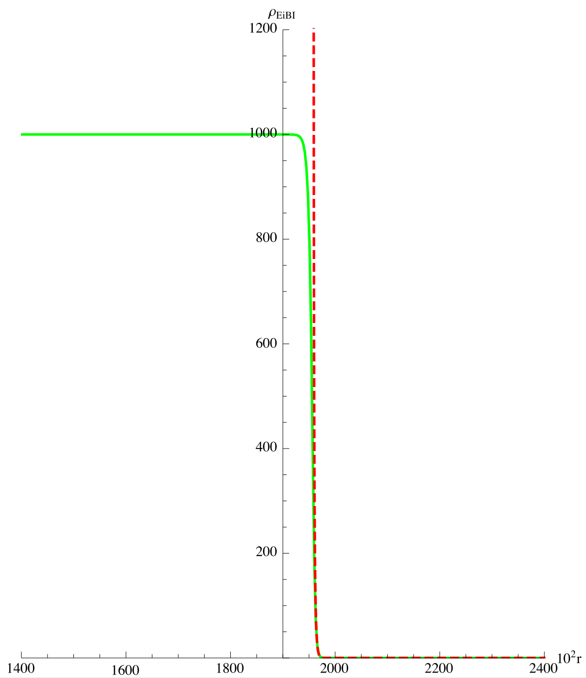
Effective Energy Density $-G^t{}_t$



Canonical Energy Density $-T^t{}_t$



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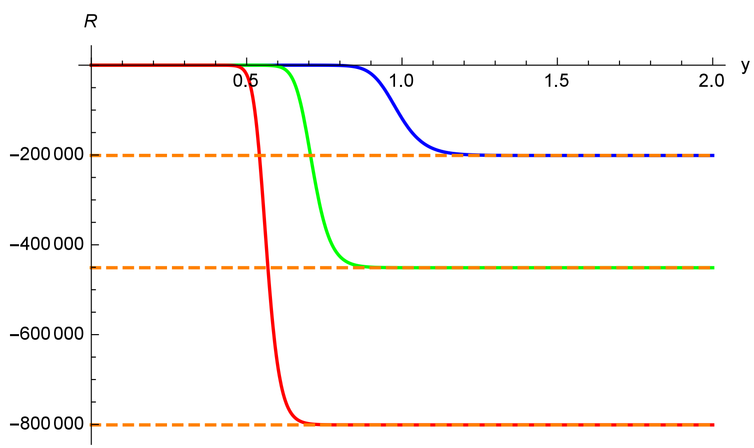
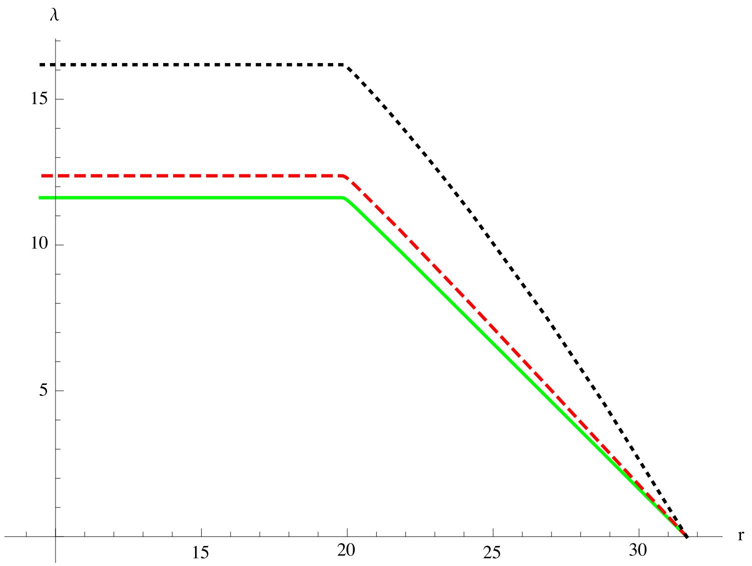
Ricci Scalar



Radial Null Geodesics



- In GR $R \sim e^{|\beta| y}$, while in EiBI with $\epsilon < 0$ we have $R \sim e^{\frac{\kappa^2 y}{|\beta|}}$.

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- Ricci-Based Gravity theories (RBGs) in the **Palatini formulation**:
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  - Degeneracies btwn modified gravity and GR with exotic sources may arise.
- Conclusion:

> There might be more exotic compact objects than our imagination can grasp. Generating new solutions may help us parametrize the space of potential deviations and their implications

Thanks !!!