Compact Objects in Gravity Theories

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Outline

1. Introduction
2. Neutron Stars
   - GR
   - Beyond GR
3. Black Holes
4. Conclusions

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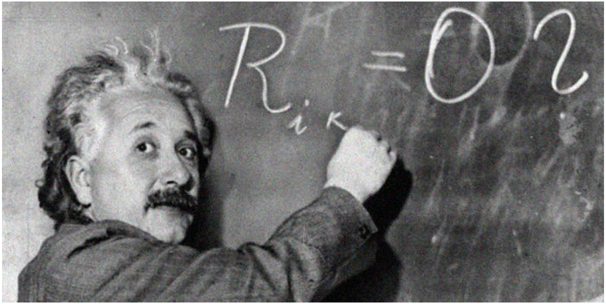
Introduction

General Relativity



- Incompatibility with Quantum Mechanics
- Singularities
- Dark Matter, Dark Energy
- ...

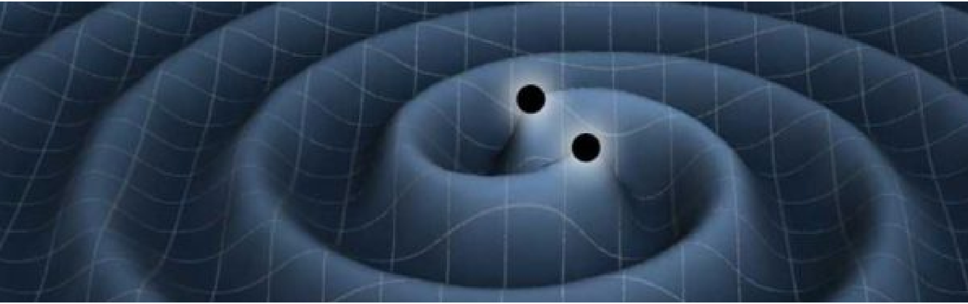
Introduction

GR or Alternative Theories of Gravity

- Scalar-tensor theories
- $f(R)$ theories
- Quadratic gravity (EsGB, CS, ...)
- ...

Introduction

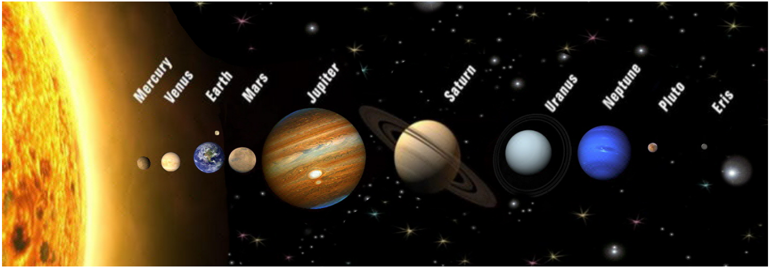
GR or Alternative Theories of Gravity



- Scalar-tensor theories
- $f(R)$ theories
- Quadratic gravity (EsGB, CS, ...)
- ...

Introduction

GR or Alternative Theories of Gravity



- Compatible with all solar system tests!
- Observational consequences: strong gravity?
  - Neutron stars
  - Black holes
  - Exotic objects

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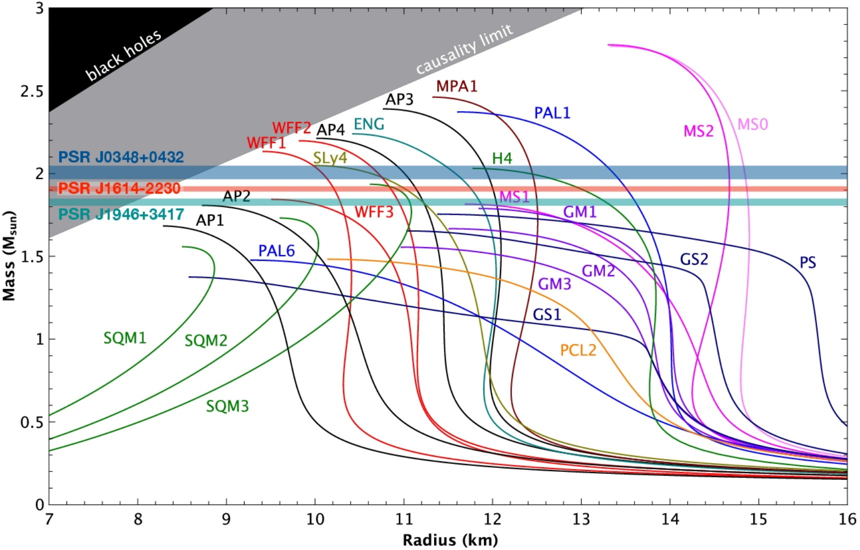
Outline

Neutron stars

What is the equation of state?

Neutron stars



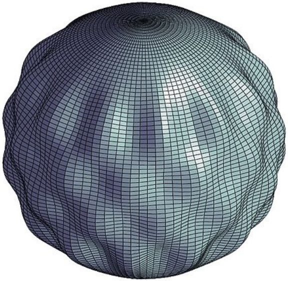
https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Neutron stars

Universal relations

Relations between neutron star properties, that are to a large extent independent of the neutron star's internal structure (EOS).

- moment of interia I
- quadrupole moment Q
- Love number
- ...



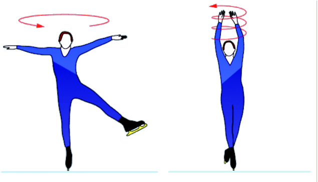
- quasi-normal modes: asteroseismology

no exact relations, but valid at the (few) percent level

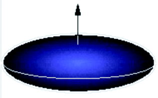
Universal relations

Yagi and Yunes I-Love-Q

moment of inertia

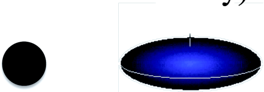


Q (spin-induced) quadrupole moment



λ_2 tidal Love number

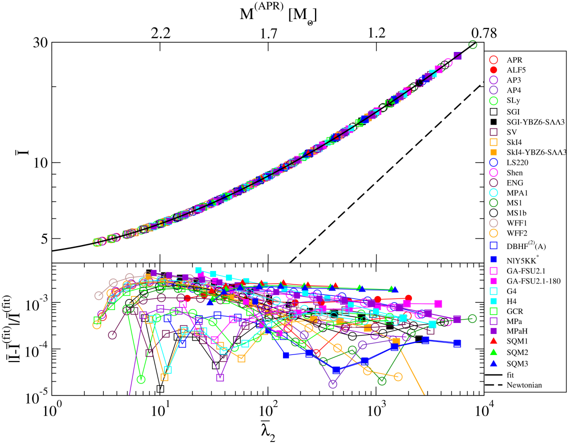
(tidal deformability)



$$\lambda_2 = \frac{\text{(tidally induced) Q}}{\text{tidal field}}$$

Universal relations

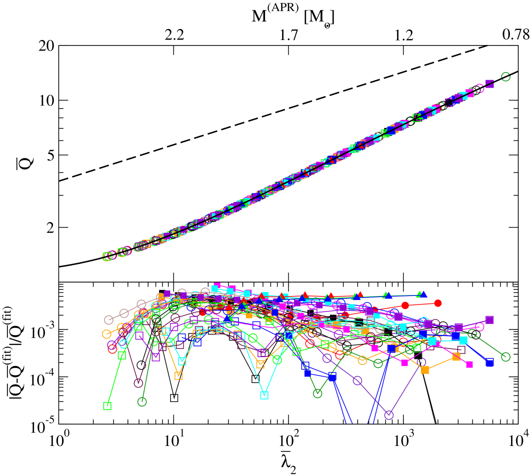
Yagi et al. 1302.4499, 1608.02582



I-Love relation: $\bar{I} = I/M^3$, $\bar{\lambda}_2 = \lambda_2/M^5$

Universal relations

Yagi et al. 1302.4499, 1608.02582

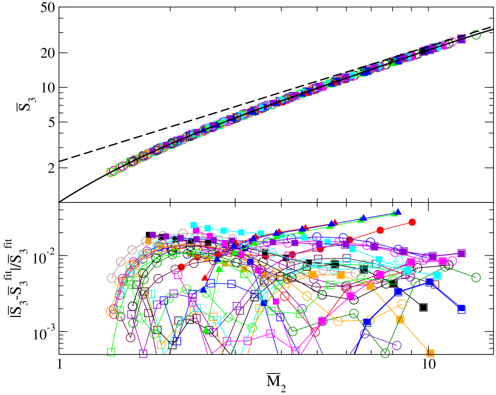


Q-Love relation: $\bar{Q} = -QM/J^2$, $\bar{\lambda}_2 = \lambda_2/M^5$

Universal relations

Yagi et al. 1608.02582, 1312.4532

three hair relations: relativistic results



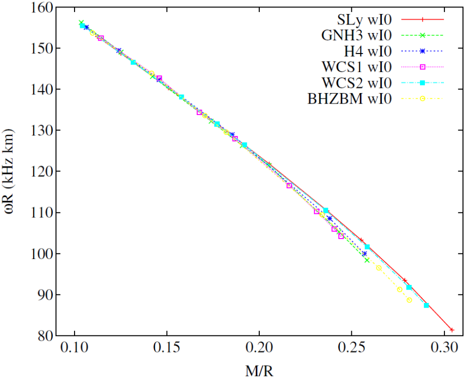
universal $\bar{S}_3$–$\bar{M}_2$ relation: $\sim 4\%$ fit (solid), Newton (dashed)

Universal relations

Blazquez-Salcedo et al. arXiv:1307.1063

quasi-normal modes: polar (parity even)

$\omega = \omega_R + i\omega_I$, frequency ω_R, damping time $\tau = 1/\omega_I$



universal $\omega_R R$–M/R relation

Universal relations

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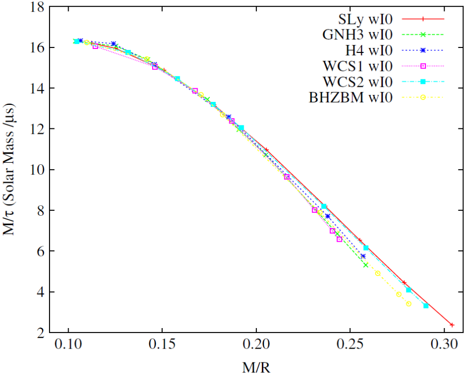


universal $M\omega_I$–M/R relation

Outline

Scalar-Tensor Theories

action: Jordan frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left[F(\Phi)\tilde{\mathcal{R}} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right] + S_m \left[\Psi_m; \tilde{g}_{\mu\nu}\right]$$

action: Einstein frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} \left[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi) \right] + S_m[\Psi_m; \mathrm{A}^2(\varphi) g_{\mu\nu}]$$

relations between the Jordan frame functions $F(\Phi)$ and $U(\Phi)$ and the Einstein frame functions $A(\varphi)$ and $V(\varphi)$

$$A(\varphi) = F^{-1/2}(\Phi) \ , \qquad 2V(\varphi) = U(\Phi)F^{-2}(\Phi)$$

Scalar-Tensor Theories

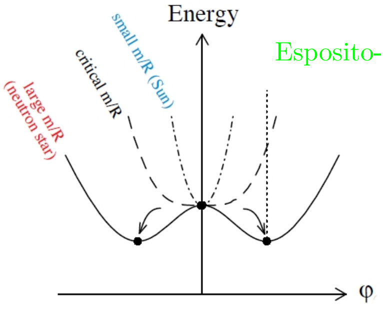
VOLUME 70, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1993

Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation

Thibault Damour
Institut des Hautes Etudes Scientifiques, 91440 Bures sur Yvette, France
and Département d'Astrophysique Relativiste et de Cosmologie, Observatoire de Paris,
Centre National de la Recherche Scientifique, 92195 Meudon, France

Gilles Esposito-Farèse
Centre de Physique Théorique, Centre National de la Recherche Scientifique, Luminy, Case 907,
13288 Marseille CEDEX 9, France

Esposito-Farese (lecture)

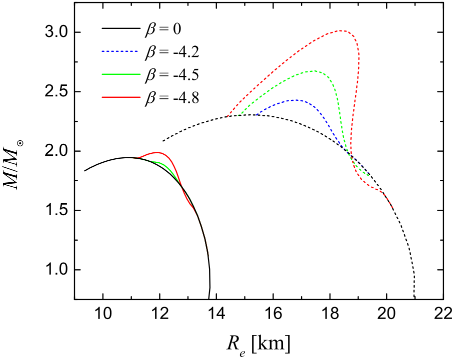


"spontaneous scalarization"

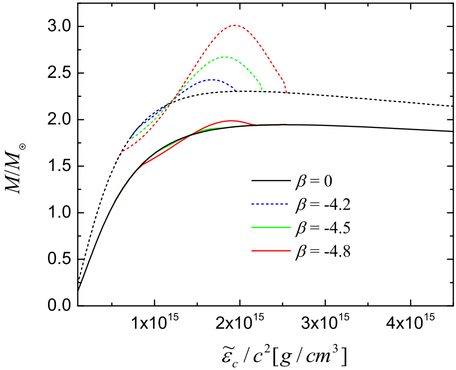
Scalar-Tensor Theories

Doneva et al. 1309.0605

$\ln A(\varphi) = \frac{1}{2}\beta_0\varphi^2$, $\beta_0 > -4.5$



mass–radius

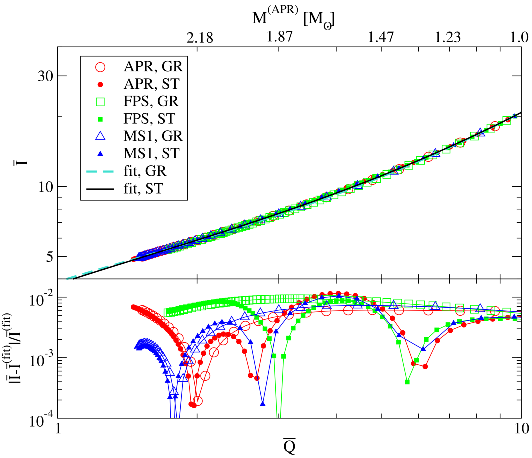


mass–energy density

spontaneous scalarization: static and Kepler limit

Scalar-Tensor Theories

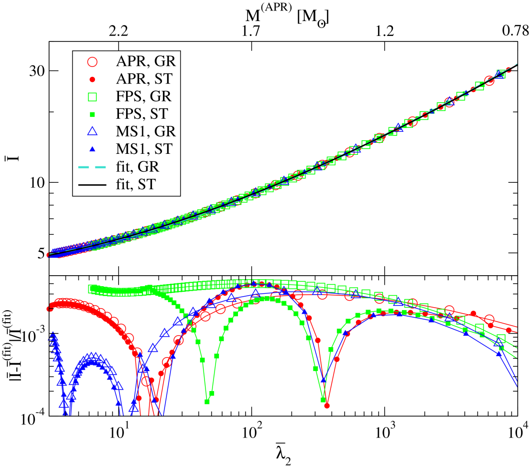
Pani et al. 1405.4547, Yagi et al. 1608.02582



I–Q: slow rotation (2nd order, $\beta_0 = -4.5$)

Scalar-Tensor Theories

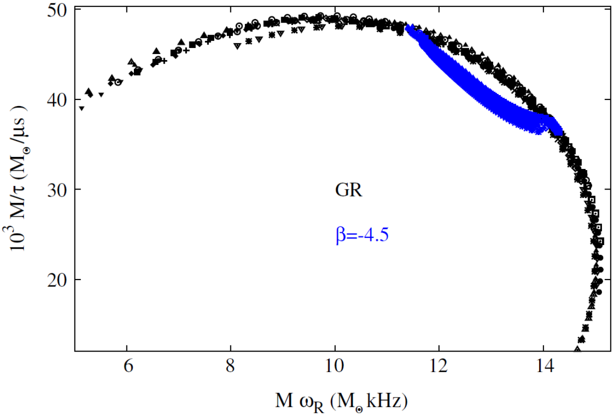
Pani et al. 1405.4547, Yagi et al. 1608.02582



I–Love: slow rotation (2nd order, $\beta_0 = -4.5$)

Scalar-Tensor Theories

Motahar et al. arXiv:1807.02598

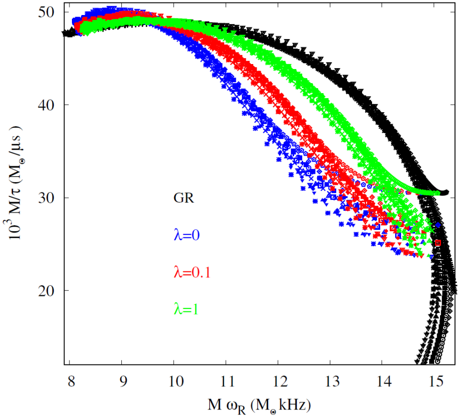


axial (odd parity) quasi-normal modes: $M\omega_I$–$M\omega_R$ ($\beta_0 = -4.5$)

Scalar-Tensor Theories

Motahar et al. arXiv:1902.01277

$$V(\varphi) = 2m_\varphi^2 + \lambda\varphi^4 \qquad m_\varphi \sim 10^{-12}\ \text{eV}$$



axial (odd parity) quasi-normal modes: $M\omega_I$–$M\omega_R$ $(\beta_0 = -6)$

Scalar-Tensor Theories

Horndeski gravity

second-order field equations and one scalar field

$$S = \int d^4x\sqrt{-g}\Big\{K(\phi, X) - G_3(\phi, X)\Box\phi$$
$$+ G_4(\phi, X)R + G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$
$$+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}(\phi, X)}{6}\big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)$$
$$+ 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla_\sigma\phi)(\nabla^\nu\nabla^\sigma\phi)\big]\Big\},$$

K and G_i's ($i = 1 \ldots 5$):
functions of the scalar field ϕ and of its kinetic term $X = -1/2\partial^\mu\phi\partial_\mu\phi$
$G_{i,X}$:
derivatives of G_i with respect to X

Scalar-Tensor Theories

Charmousis et al. 1106.2000, 1112.4866

subsector: Fab Four

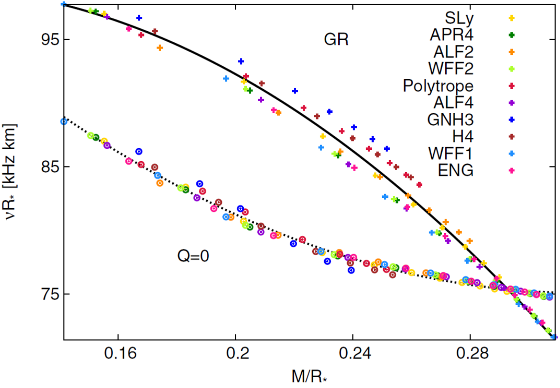
special cases of Horndeski gravity

- general relativity (“George”)
- Einstein-dilaton-Gauss-Bonnet gravity (“Ringo”)
- theories with a nonminimal coupling with the Einstein tensor (“John”)
- theories involving the double-dual of the Riemann tensor (“Paul”)

Scalar-Tensor Theories

Blazquez-Salcedo et al. arXiv:1803.01655

"George" + "John" $\phi(r,t) = Qt + F(r)$



axial (odd parity) quasi-normal modes: $\omega_R R$–M/R

Scalar-Tensor Theories

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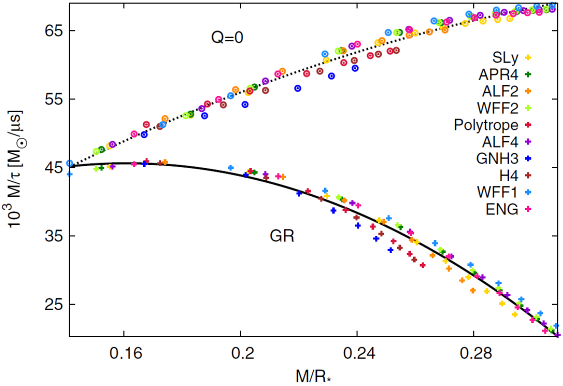


axial (odd parity) quasi-normal modes: $\omega_I M$–M/R

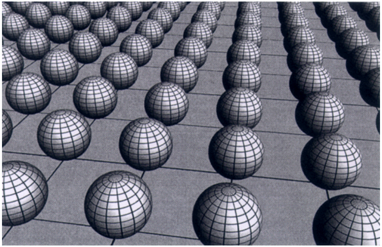
Einstein-Gauss-Bonnet-Dilaton Theory

Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{4} e^{-\gamma\phi} R^2_{\mathrm{GB}} \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\mathrm{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$



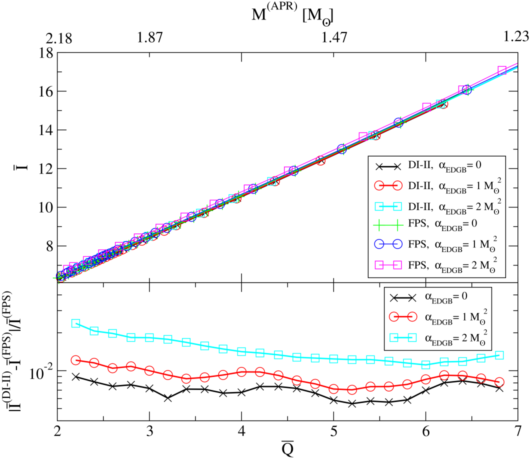
- α Gauss-Bonnet coupling constant
- γ dilaton coupling constant ($\gamma = 1$)

Bounds

- observational bound: BH low-mass X-ray binaries: $\sqrt{\alpha} \lesssim 3.8 \times 10^5 \mathrm{cm}$
- theoretical lower bound on BH mass: $\dfrac{\alpha}{M^2} \lesssim 0.691$

Einstein-Gauss-Bonnet-Dilaton Theory

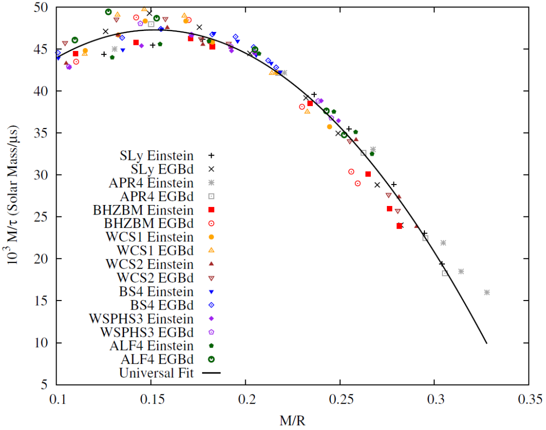
Kleihaus et al. 1601.05583 , Yagi et al. 1608.02582



$\bar{I}$–$\bar{Q}$: rapid rotation

Einstein-Gauss-Bonnet-Dilaton Theory

Blazquez-Salcedo et al. arXiv:1511.03960



axial (odd parity) quasi-normal modes: $M\omega_I$–M/R

Chern-Simons Gravity

action

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left[R - 2\nabla_\mu \phi \nabla^\mu \phi - V(\phi) + \alpha_{\rm CS}\, \phi\, {}^*RR \right]$$

two cases

- dynamical:

  scalar true dynamical degree of freedom

  dCS gravity
- nondynamical:

  scalar kinetic term absent

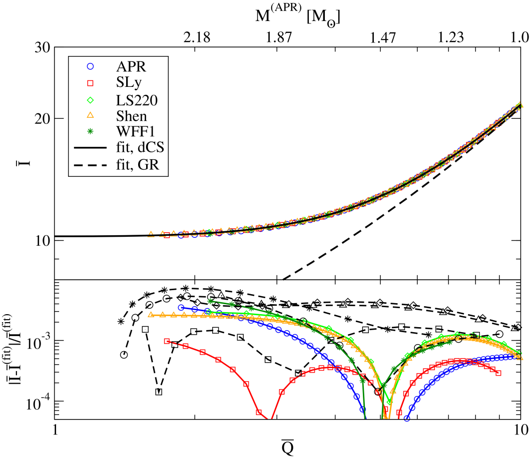
- a spherically symmetric solution of GR is also a solution of dCS gravity
- corrections in the presence of a parity-odd source such as rotation

bound: Gravity Probe B

$$\sqrt{|\alpha_{\rm CS}|} < \mathcal{O}(10^{13}) {\rm cm}$$

Chern-Simons Gravity

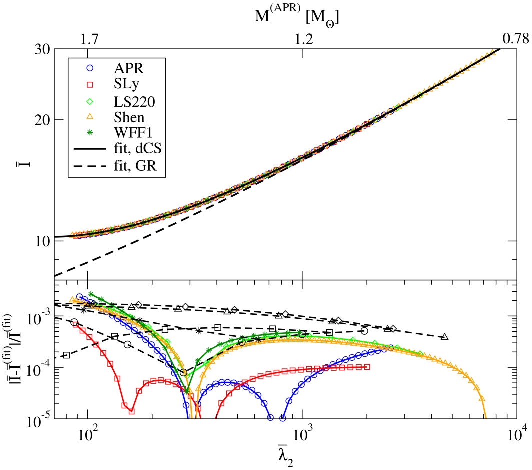
Yagi et al. 1608.02582



$\bar{I}$–$\bar{Q}$: slow rotation

Chern-Simons Gravity

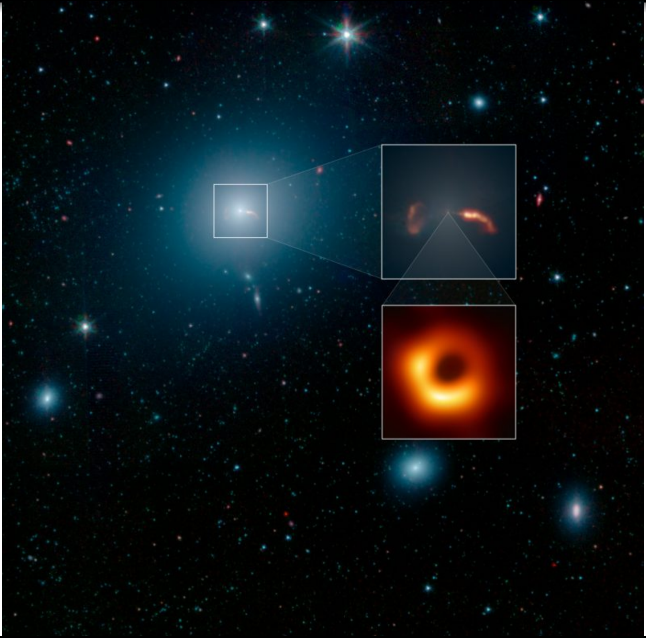
Yagi et al. 1608.02582



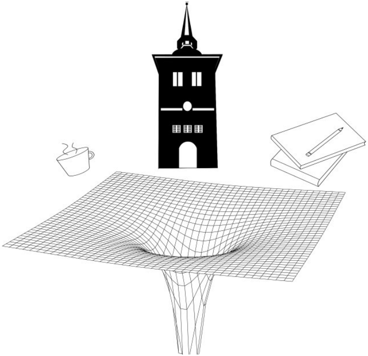
$\bar{I}$–$\bar{\lambda}_2$: slow rotation

Outline

Black holes in GR



Black holes in GR



A Kerr black hole has no hair

A Kerr black hole is fully characterized in terms of only two global parameters: the mass M and the angular momentum J

Black holes in GR

Geroch, J. Math. Phys. (1970); Hansen, J. Math. Phys. (1974); Thorne, Rev. Mod. Phys. (1980)

Multipole moments M_l (g_{00}) and S_l ($g_{0\phi}$)

All multiple moments can be expressed in terms of only two quantities

$$M_0 = M \qquad\qquad S_1 = J$$

$$M_l + iS_l = M\left(i\frac{J}{M}\right)^l$$

Quadrupole moment

$$M_2 = Q = -\frac{J^2}{M}$$

Black holes in GR

Grenzebach et al. arXiv:1403.5234

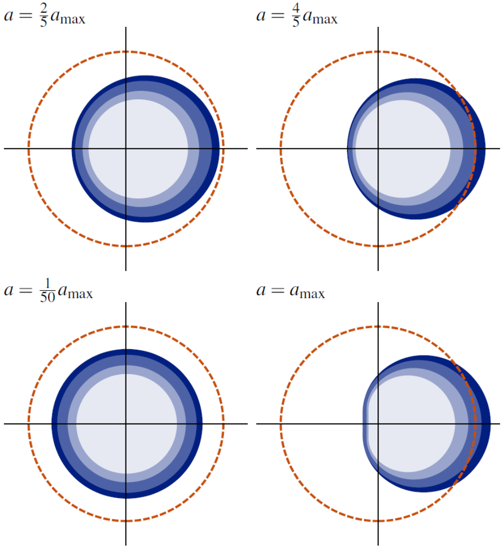
angular momentum bound

$$j = \frac{J}{M^2} \leq 1$$

- < 1 non-extremal black hole
- $= 1$ extremal black hole
- > 1 naked singularity

shadow

β	0	0	$\frac{5}{9}m^2$	0
ℓ	0	$\frac{3}{4}m$	$\frac{4}{3}m$	$\frac{4}{3}m$
Λ	0	0	$10^{-2}m^{-2}$	0
$a_{\max}$	m	$\frac{5}{4}m$	$1.51m$	$\frac{5}{3}m$
	Kerr	Kerr–NUT	KN–NUT with Λ	Kerr–NUT



EGBd black holes

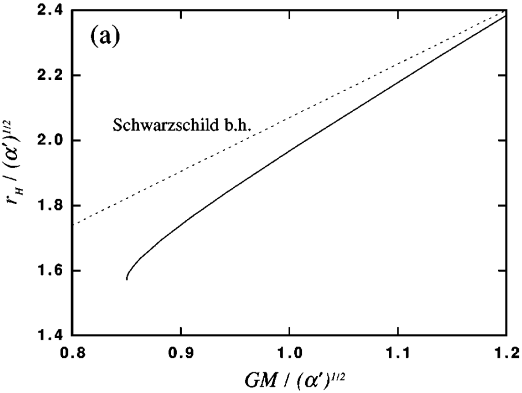
Kanti et al. arXiv:hep-th/9511071, Torii et al. arXiv:gr-qc/9606034

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4}e^{2\phi_h}}$$

lower bound
on the horizon size
for fixed α'

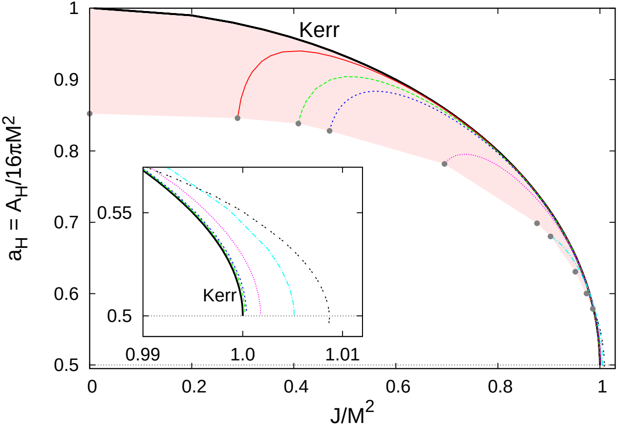


lower bound on the mass

EGBd black holes

Kleihaus et al. arXiv:1101.2868

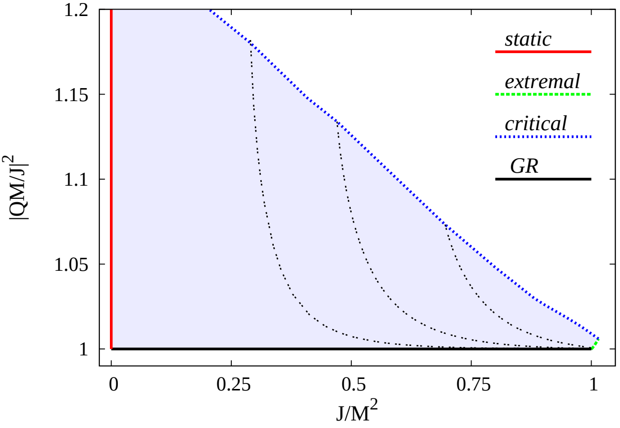
horizon area versus angular momentum



EGBd black holes

Kleihaus et al. arXiv:1101.2868

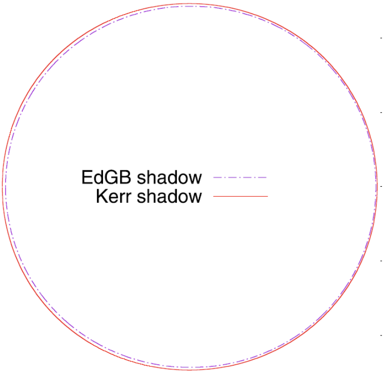
quadrupole moment versus angular momentum



EGBd black holes

Cunha et al. arXiv:1701.00079

shadow



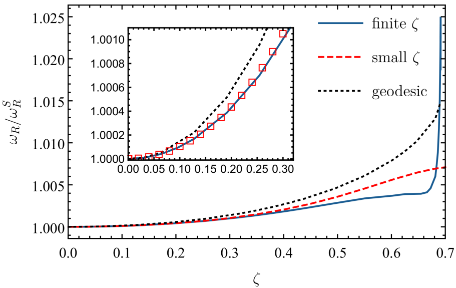
$$\alpha/M^2 = 0.172,\ J/M^2 = 0.41$$

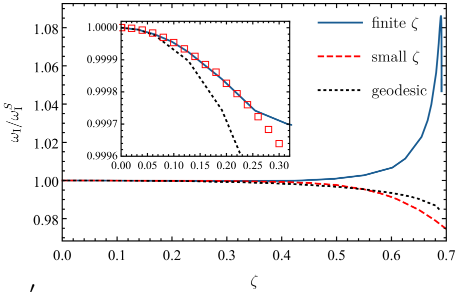
EGBd black holes

Blazquez-Salcedo et al. arXiv:1609.01286

quasi-normal mode (axial $l = 2$) versus coupling constant

normalized to the Schwarzschild values





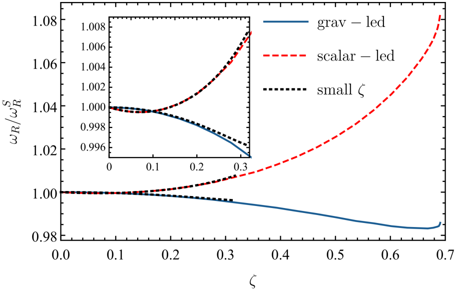
real part $\qquad \zeta = \dfrac{\alpha'}{M^2} \qquad$ imaginary part

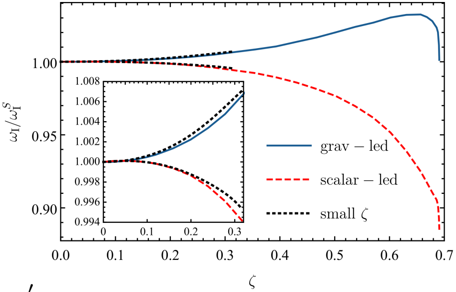
EGBd black holes

Blazquez-Salcedo et al. arXiv:1609.01286

quasi-normal mode (polar $l = 2$) versus coupling constant

normalized to the Schwarzschild values





real part $\qquad \zeta = \frac{\alpha'}{M^2} \qquad$ imaginary part

EsGB black holes

Doneva et al. arXiv:1711.01187, Silva et al. arXiv:1711.02080, Antoniou et al. arXiv:1711.03390

Curvature induced scalarized black holes

action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}^2_{GB} \Big]$$

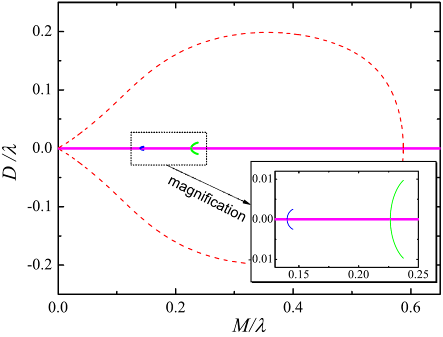
coupling function

$$f(\varphi) = \frac{1}{12} \left(1 - e^{-6\varphi^2} \right)$$

small φ

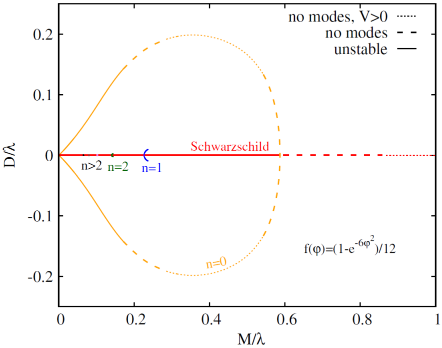
$$f(\varphi) = \frac{1}{2} \varphi^2$$

sequence of radial excitations



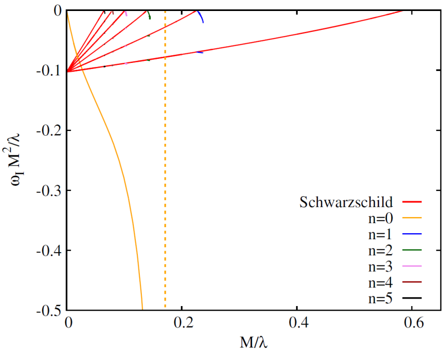
EsGB black holes

Blazquez-Salcedo et al. arXiv:1805.05755



unstable modes

Schwarzschild red
scalarized $n = 0$ orange
scalarized $n > 0$ ...

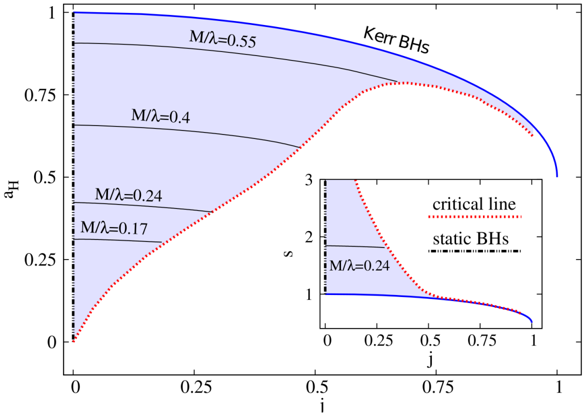


solutions

Schwarzschild red
scalarized $n = 0$ orange
scalarized $n > 0$ ...

EsGB black holes

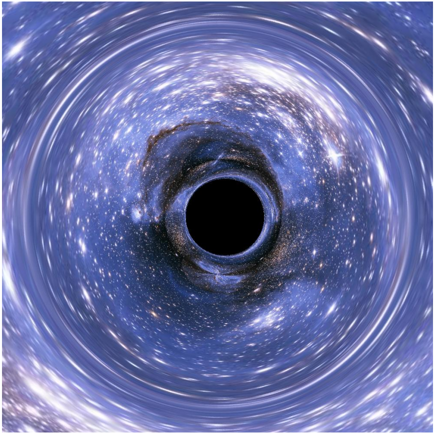
Cunha et al. arXiv:1904.09997



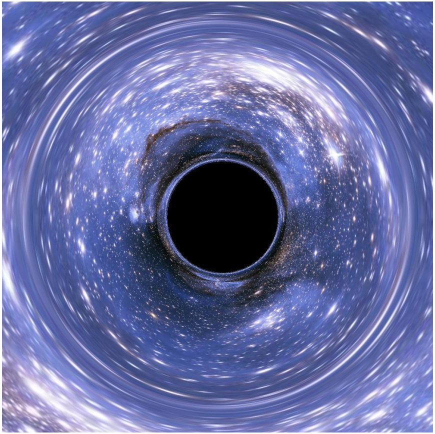
rotating EsGB black holes

EsGB black holes

Cunha et al. arXiv:1904.09997



EsGB



Kerr

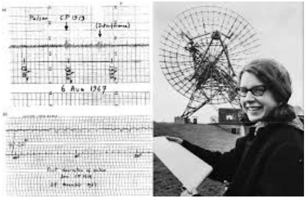
$$M/\lambda = 0.237 (j = 0.24)$$

Outline

Conclusions

GR versus generalized gravity theories

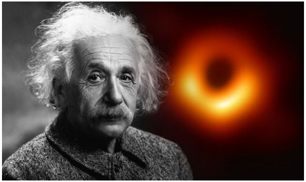
neutron stars



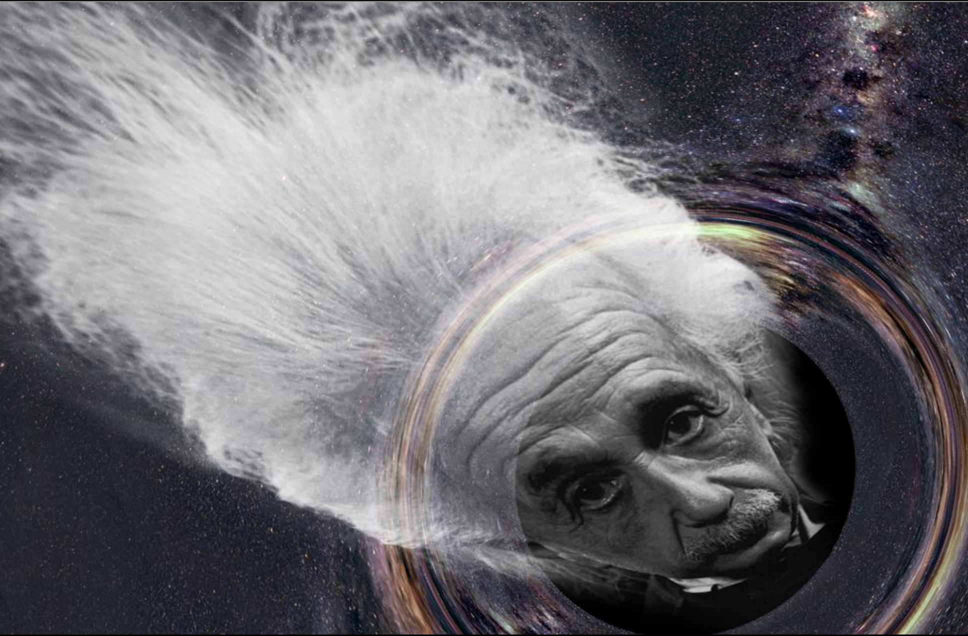
- properties
- quasi-normal modes
- universal relations
- ...

black holes

- properties
- shadow
- quasi-normal modes
- ...



THANKS



Quasinormal Modes

Static Spherically Symmetric Backgrounds

Metric

$$ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -F(r)dt^2 + K(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Scalar

$$\phi = \phi_0(r).$$

Matter

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$p = p_0(r), \quad \rho = \rho_0(r), \quad u = u^{(0)} = u^t \partial_t$$

$u^2 = -1$

Equation of state

$$\rho = \rho(p)$$

Quasinormal Modes

Perturbations

Metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$

Scalar

$$\phi = \phi_0(r) + \epsilon\delta\phi(t, r, \theta, \varphi)$$

Matter

$$\begin{aligned} p &= p_0(r) + \epsilon\delta p(t, r, \theta, \varphi) \\ \rho &= \rho_0(r) + \epsilon\delta\rho(t, r, \theta, \varphi) \\ u &= u^{(0)} + \epsilon\delta u(t, r, \theta, \varphi) \end{aligned}$$

Quasinormal Modes

Axial modes: odd-parity perturbations

$$Y_{lm}(\theta,\varphi) \to Y_{lm}(\pi-\theta,\pi+\varphi) = (-1)^{l+1} Y_{lm}(\theta,\varphi)$$

Polar modes: even-parity perturbations

$$Y_{lm}(\theta,\varphi) \to Y_{lm}(\pi-\theta,\pi+\varphi) = (-1)^{l} Y_{lm}(\theta,\varphi)$$

Eigenvalue ω

$$\omega = \omega_R + i\omega_I$$

Frequency: ω_R
Decay time: $\tau = -1/\omega_I$

Stable perturbations:

$$\omega_I < 0$$

Unstable perturbations:

$$\omega_I > 0$$

Quasinormal Modes

Axial channel

Metric

$$h_{\mu\nu}^{(\mathrm{axial})} = \int d\omega \sum_{l,m} e^{-i\omega t}$$

$$\times \begin{bmatrix} 0 & 0 & -h_0 \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}Y_{lm} & h_0 \sin\theta \frac{\partial}{\partial\theta}Y_{lm} \\ 0 & 0 & -h_1 \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}Y_{lm} & h_1 \sin\theta \frac{\partial}{\partial\theta}Y_{lm} \\ -h_0 \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}Y_{lm} & -h_1 \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}Y_{lm} & h_2 \frac{1}{2\sin\theta}X_{lm} & -\frac{1}{2}h_2 \sin\theta\, W_{lm} \\ h_0 \sin\theta \frac{\partial}{\partial\theta}Y_{lm} & h_1 \sin\theta \frac{\partial}{\partial\theta}Y_{lm} & -\frac{1}{2}h_2 \sin\theta\, W_{lm} & -\frac{1}{2}h_2 \sin\theta\, X_{lm} \end{bmatrix}$$

$$X_{lm} = 2\frac{\partial^2}{\partial\theta\,\partial\varphi}Y_{lm} - 2\cot\theta\frac{\partial}{\partial\varphi}Y_{lm}\,, \quad W_{lm} = \frac{\partial^2}{\partial\theta^2}Y_{lm} - \cot\theta\frac{\partial}{\partial\theta}Y_{lm} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Y_{lm}$$

Scalar, matter

$$\delta\phi = \delta\rho = \delta p = \delta u_\mu^{(\mathrm{axial})} = 0$$

Quasinormal Modes

Polar channel

Metric

$$
\begin{aligned}
h_{\mu\nu}^{(\mathrm{polar})} &= \int d\omega \sum_{l,m} e^{-i\omega t} \\
&\times \begin{bmatrix} 2NFY_{lm} & -H_1 Y_{lm} & -h_{0p}\frac{\partial}{\partial\theta}Y_{lm} & -h_{0p}\frac{\partial}{\partial\varphi}Y_{lm} \\ -H_1 Y_{lm} & -2KLY_{lm} & h_{1p}\frac{\partial}{\partial\theta}Y_{lm} & h_{1p}\frac{\partial}{\partial\varphi}Y_{lm} \\ -h_{0p}\frac{\partial}{\partial\theta}Y_{lm} & h_{1p}\frac{\partial}{\partial\theta}Y_{lm} & B & -r^2 V X_{lm} \\ -h_{0p}\frac{\partial}{\partial\varphi}Y_{lm} & h_{1p}\frac{\partial}{\partial\varphi}Y_{lm} & -r^2 V X_{lm} & A \end{bmatrix}
\end{aligned}
$$

$$A = (l(l+1)V - 2T)r^2 \sin^2\theta\, Y_{lm} + r^2 V \sin^2\theta\, W_{lm}$$

$$B = (l(l+1)V - 2T)r^2 Y_{lm} - r^2 V W_{lm}$$

Quasinormal Modes

Polar channel

Scalar

$$\delta\phi^{(\text{polar})} = \int d\omega \sum_{l,m} e^{-i\omega t} \Phi_1 \, Y_{lm}$$

Matter

$$\begin{aligned} \delta\rho^{(\text{polar})} &= \int d\omega \sum_{l,m} e^{-i\omega t} \rho_1 \, Y_{lm} \\ \delta p^{(\text{polar})} &= \int d\omega \sum_{l,m} e^{-i\omega t} p_1 \, Y_{lm} \end{aligned}$$

$$\delta u^{\mu}_{(\text{polar})} = \int d\omega \sum_{l,m} e^{-i\omega t} \left(\frac{-N}{\sqrt{-F}} Y_{lm}, W_f Y_{lm}, V_f \frac{\partial}{\partial\theta} Y_{lm}, V_f \frac{\partial}{\partial\varphi} Y_{lm} \right)$$

Quasinormal Modes

Axial case: h_0, h_1

generalization of Regge-Wheeler equation

Polar case: $H_1, T, p_1, V_f, \Phi_1, \frac{d}{dr}\Phi_1$

generalization of Zerilli equation

- scalar-led modes
- gravitational-led modes

Asymptotic behaviour: $r \to \infty$

$$\Psi \sim A_{in} e^{-i\omega(t+R^*)} + A_{out} e^{-i\omega(t-R^*)}$$

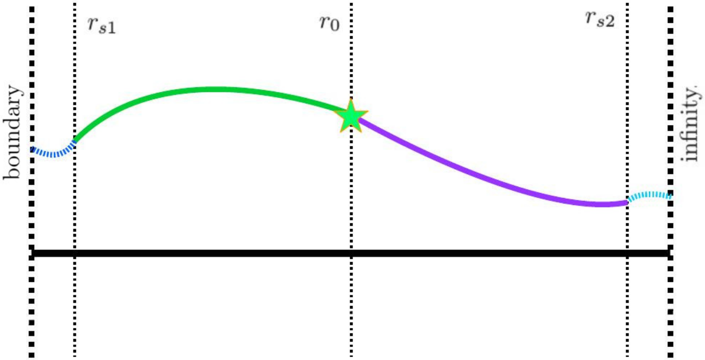
R^*: generalized tortoise coordinate

Ringdown phase of black holes and neutron stars

$$A_{in} = 0 : \quad \text{radiation away from compact object}$$

Quasinormal Modes

Calculation



Perturbative solutions: 1. boundary-r_{s1}, 2. r_{s2}-infinity
Numerical solutions: 1. r_{s1}-r_0, 2. r_0-r_{s2}
QNM frequency ω: matching at r_0

Scalar-Tensor Theories

action: Jordan frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left[F(\Phi)\tilde{\mathcal{R}} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right] + S_m\left[\Psi_m; \tilde{g}_{\mu\nu}\right]$$

G_*: gravitational constant

$\tilde{\mathcal{R}}$: Ricci scalar with respect to $\tilde{g}_{\mu\nu}$

Φ: gravitational scalar field

S_m: matter action

Ψ_m: matter fields

Φ does not couple directly to Ψ_m: weak equivalence principle is satisfied

Scalar-Tensor Theories

transformation to Einstein frame

$$\left(\frac{d\varphi}{d\Phi}\right)^2 = \frac{3}{4}\left(\frac{d\ln(F(\Phi))}{d\Phi}\right)^2 + \frac{Z(\Phi)}{2F(\Phi)}$$

action: Einstein frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)] + S_m[\Psi_m; \mathrm{A}^2(\varphi)g_{\mu\nu}]$$

relations between the Jordan frame functions $F(\Phi)$ and $U(\Phi)$ and the Einstein frame functions $A(\varphi)$ and $V(\varphi)$

$$A(\varphi) = F^{-1/2}(\Phi)\ , \qquad 2V(\varphi) = U(\Phi)F^{-2}(\Phi)$$

Scalar-Tensor Theories

Brans–Dicke Theory

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left[\Phi \tilde{\mathcal{R}} - \frac{\omega(\Phi)}{\Phi} \tilde{g}^{\mu\nu} \left(\partial_\mu \Phi\right) \left(\partial_\nu \Phi\right) - 2U(\Phi) \right] + S_m[\Psi_m, \tilde{g}_{\mu\nu}]$$

relation between Jordan-frame and Einstein-frame quantites

$$\Phi = A^{-2}(\varphi) \ , \quad 3 + 2\omega(\Phi) = \alpha(\varphi)^{-2}$$

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$\alpha(\varphi) = \alpha_0$ =constant, i.e., $\omega(\Phi)$ =constant
observational bound: $\omega > 40000$ (Cassini-Huygens)
limit: $\omega \to \infty$ GR

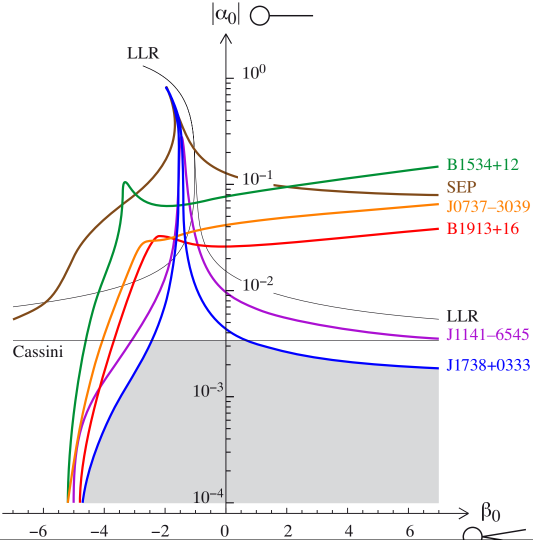
Scalar-Tensor Theories

Freire et al. 1205.1450

observational constraints on STT parameters

$$\ln A(\varphi) = \ln A(\varphi_0)$$
$$+\alpha_0(\varphi - \varphi_0)$$
$$+\frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

$m_\varphi = 0$



Quadratic Gravity

Curvature invariants

$$R^2\,, \quad R^2_{\mu\nu}\,, \quad R^2_{\mu\nu\rho\sigma}\,, \quad {}^*RR$$

$$R^2_{\mu\nu} \equiv R_{\mu\nu}R^{\mu\nu}$$

Kretschmann scalar

$$R^2_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Pontryagin/Chern-Simons scalar

$${}^*RR \equiv \frac{1}{2} R_{\mu\nu\rho\sigma}\epsilon^{\nu\mu\lambda\kappa}R^{\rho\sigma}{}_{\lambda\kappa}$$

Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$

Gauss-Bonnet scalar

$$R^2_{GB} \equiv R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}$$

Einstein-Gauss-Bonnet-Dilaton Theory

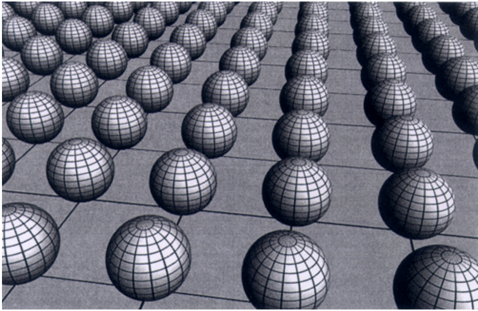
String Theory

unification of all fundamental interactions

dimensional reduction to 4 spacetime dimensions:

low energy effective theories

- additional fields
  - dilaton
  - axion
  - Maxwell fields
  - Yang-Mills fields
  - ...
- higher order curvature corrections
  - Gauss-Bonnet term
  - ...
- ...



Einstein-Gauss-Bonnet-Dilaton Theory

Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \phi)^2 + \frac{\alpha}{4} e^{-\gamma\phi} R^2_{\rm GB} \right]$$

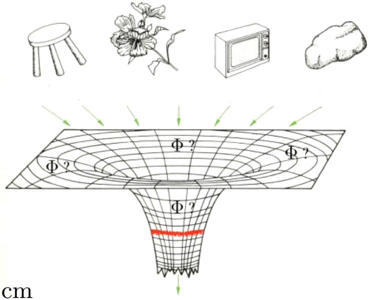
Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

- α Gauss-Bonnet coupling constant
- γ dilaton coupling constant ($\gamma = 1$)

In 4 spacetime dimensions the coupling to the dilaton is needed.
The resulting set of equations of motion are of second order.

Einstein-Gauss-Bonnet-Dilaton Theory



consequences

- scalar "hair": dilaton "hair"
- negative energy density

bounds on α $(\gamma = 1)$

- observational
  - Shapiro time delay

$$\sqrt{\alpha} \lesssim 10^{13}\mathrm{cm}$$

  - BH low-mass X-ray binaries

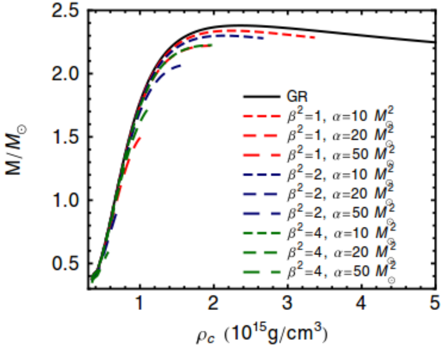
$$\sqrt{\alpha} \lesssim 3.8 \times 10^{5}\mathrm{cm}$$

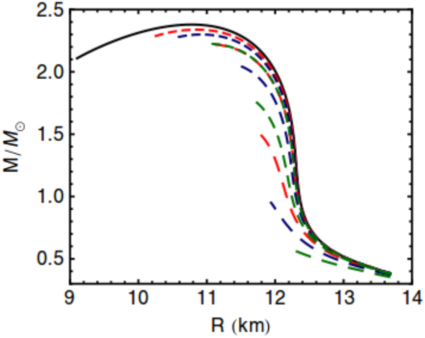
- theoretical/observational
  - lower bound on BH mass

$$\frac{\alpha}{M^2} \lesssim 0.691$$

Einstein-Gauss-Bonnet-Dilaton Theory

Pani et al. 1109.0928





static neutron stars with APR EoS: dependence on α and β

branches end

expansion around origin: square roots

reality condition: condition on $\alpha\beta$, maximum central density