

Solar system consistency of teleparallel gravity

Post-Newtonian limit and PPN parameters of various models

Manuel Hohmann

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence “The Dark Side of the Universe”



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- 1 Introduction
- 2 Parametrized post-Newtonian formalism
- 3 Teleparallel geometry and tetrad PPN formalism
- 4 PPN limits of teleparallel theories
 - Generic $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ teleparallel gravity
 - Scalar-torsion extension of scalar-curvature gravity
 - Generic $L(T, X, Y, \phi)$ scalar-torsion gravity
- 5 Conclusion

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 - Accelerating expansion of the universe.
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 - Corner of “Geometrical Trinity of Gravity” [Beltrán Jiménez, Heisenberg, Koivisto '19]
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↪ Theories considered in this talk:
 1. Generic $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ teleparallel gravity.
 2. Scalar-torsion analogue and extension of scalar-curvature gravity.
 3. Generic $L(X, T, Y, \phi)$ scalar-torsion gravity.

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- Energy-momentum tensor of a perfect fluid:

$$\Theta^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + pg^{\mu\nu}.$$

- Rest mass density ρ .
- Specific internal energy Π .
- Pressure p .
- Four-velocity u^μ .

Post-Newtonian matter and velocity orders

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 - Consider particular coordinates (“universe rest frame”).
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- Energy-momentum tensor of a perfect fluid:

$$\Theta^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + pg^{\mu\nu}.$$

- Rest mass density $\rho \sim \mathcal{O}(2)$.
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- Assign velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$ to all quantities.

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- Assign velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$ to all quantities.
- Quasi-static: assign additional $\mathcal{O}(1)$ to time derivatives ∂_0 .

- Standard post-Newtonian metric expansion:

$$g_{\mu\nu} = g_{\mu\nu}^0 + g_{\mu\nu}^1 + g_{\mu\nu}^2 + g_{\mu\nu}^3 + g_{\mu\nu}^4 + \mathcal{O}(5).$$

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- Properties of standard PPN metric:
 - Second-order spatial part g_{ij}^2 is diagonal.
 - Fourth-order temporal part g_{00}^4 does not contain potential \mathfrak{B} .

PPN parameters and potentials

- Metric in standard PPN gauge:

$${}^2g_{00} = 2U,$$

$${}^2g_{ij} = 2\gamma U\delta_{ij},$$

$${}^3g_{0i} = -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i,$$

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- ↪ Perturbatively solve field equations order by order.
- ↪ Compare with PPN metric to get PPN parameters.

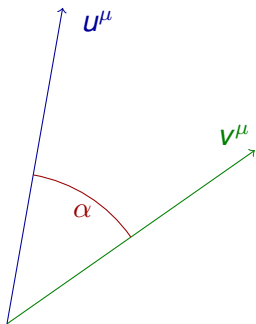
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General relativity and curvature

Metrics $g_{\mu\nu}$:

$$g_{\mu\nu} u^\mu v^\nu = \|u\| \|v\| \cos \alpha.$$

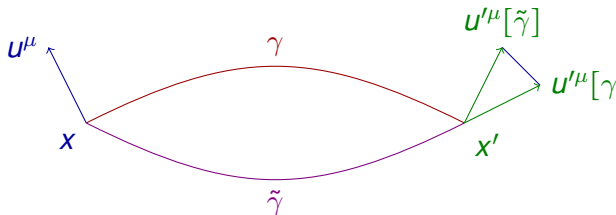
- Vectors u^μ, v^ν .
- Lengths $\|u\|, \|v\|$.
- Angle α .



Connection and parallel transport ∇ :

- Moves vector u^μ from x to x' .
- Motion follows trajectory γ .
- Result depends on the trajectory.
- Curvature: difference between results.
- Curvature mediates gravity:

$$S = \int_M \sqrt{-g} R d^4x.$$

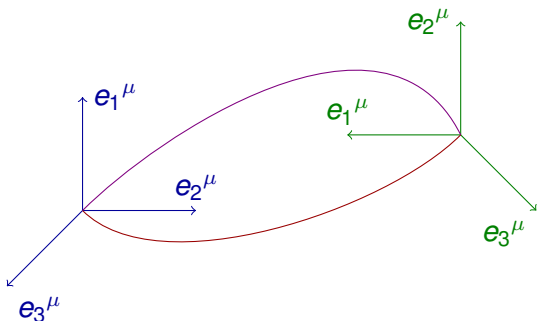


Parallel transport and teleparallel geometry

Tetrad describes geometry:

- Frame e_a^μ at every point.
- Frames generate transport.
- Independent of trajectory.

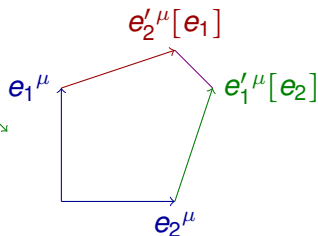
⇒ There is no curvature.



Torsion instead of curvature:

- Non-symmetric transport.
- Torsion \leftrightarrow gravity.
- Gravitational action:

$$S = \frac{1}{2} \int_M H \wedge T.$$



- Fundamental fields:

- Coframe field $\theta^A = \theta^A{}_{\mu} dx^{\mu}$.
- Flat spin connection $\omega^A{}_B = \omega^A{}_{B\mu} dx^{\mu}$.
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- Derived quantities:

- Frame field $e_A = e_A{}^\mu \partial_\mu$ with $e_A{}^\mu \theta^B{}_\mu = \delta_A^B$ and $e_A{}^\mu \theta^A{}_\nu = \delta^\mu{}_\nu$.
- Metric $g_{\mu\nu} = \eta_{AB} \theta^A{}_\mu \theta^B{}_\nu$.
- Determinant $\theta = \det(\theta^A{}_\mu)$.
- Teleparallel connection $\Gamma^\mu{}_{\nu\rho} = e_A{}^\mu (\partial_\rho \theta^A{}_\nu + \omega^A{}_{B\rho} \theta^B{}_\nu)$.
- Levi-Civita connection $\overset{\circ}{\Gamma}{}^\mu{}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} (\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho})$.

Teleparallel geometry

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- Properties of the teleparallel connection:

- Vanishing curvature:
 $R^\mu{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu{}_{\nu\sigma} - \partial_\sigma \Gamma^\mu{}_{\nu\rho} + \Gamma^\mu{}_{\tau\rho} \Gamma^\tau{}_{\nu\sigma} - \Gamma^\mu{}_{\tau\sigma} \Gamma^\tau{}_{\nu\rho} = 0$.
- Vanishing nonmetricity: $Q_{\mu\nu\rho} = \nabla_\mu g_{\nu\rho} = 0$.
- Nonvanishing torsion: $T^\mu{}_{\nu\rho} = \Gamma^\mu{}_{\rho\nu} - \Gamma^\mu{}_{\nu\rho} \neq 0$.

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⇒ Possible to use Weitzenböck gauge: $\omega^A{}_{B\mu} \equiv 0$.

- Torsion scalar:

$$T = \frac{1}{4} T^{\mu\nu\rho} T_{\mu\nu\rho} + \frac{1}{2} T^{\mu\nu\rho} T_{\rho\nu\mu} - T^{\mu}{}_{\mu\rho} T^{\nu\rho}{}_{\nu}.$$

Teleparallel equivalent of general relativity

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$$S_{\text{EH}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \overset{\circ}{R} = -\frac{1}{2\kappa^2} \int d^4x e (T - B).$$

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Post-Newtonian tetrad

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- Relation to metric components:

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Action and field equations

- Gravitational part of the action:

$$S_g[\theta, \omega] = \frac{1}{2\kappa^2} \int_M \mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) \theta \, d^4x.$$

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- PPN parameters [Ualikhanova, MH '19]:

$$\beta - 1 = -\frac{\varepsilon}{2}, \quad \gamma - 1 = -2\varepsilon, \quad \varepsilon = \frac{2F_{,1} + F_{,2} + F_{,3}}{2(2F_{,1} + F_{,2} + 2F_{,3})},$$

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- Bounds on theory parameters from Cassini tracking:

$$\gamma - 1 = -2\varepsilon = (2.1 \pm 2.3) \cdot 10^{-5}.$$

Axial-vector-tensor decomposition

- Irreducible decomposition of torsion components:

$$T_{\text{ax}} = \frac{1}{18}(\mathcal{T}_1 - 2\mathcal{T}_2), \quad T_{\text{ten}} = \frac{1}{2}(\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_3), \quad T_{\text{vec}} = \mathcal{T}_3.$$

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⇒ Purely axial modifications do not affect PPN parameters.

Particular theories

- New general relativity [Hayashi, Shirafuji '79]:
 - Most general action linear in torsion scalars:

$$\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) = t_1 \mathcal{T}_1 + t_2 \mathcal{T}_2 + t_3 \mathcal{T}_3.$$

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- $f(T)$ gravity theories [Bengochea, Ferraro '08; Linder '10]:
 - Lagrangian defined as function of linear combination:

$$\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) = f(T), \quad T = \frac{1}{4}\mathcal{T}_1 + \frac{1}{2}\mathcal{T}_2 - \mathcal{T}_3.$$

- ⇒ Taylor coefficients given by:

$$F_{,1} = \frac{1}{4}f'(0), \quad F_{,2} = \frac{1}{2}f'(0), \quad F_{,3} = -f'(0)$$

- ⇒ Indistinguishable from GR, since $\varepsilon \equiv 0$.

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- Taylor expansion of functions around background $\phi = \Phi$:

$$A = \mathcal{A}(\Phi), \quad A' = \mathcal{A}'(\Phi), \quad A'' = \mathcal{A}''(\Phi), \quad A''' = \mathcal{A}'''(\Phi), \quad \dots$$

Field equations

- Field equations:
 - Tetrad field equations - symmetric part:

$$\begin{aligned}\kappa^2 \Theta_{\mu\nu} = & (\mathcal{A}' + \mathcal{C}) \mathcal{S}_{(\mu\nu)}{}^\rho \phi_{,\rho} + \mathcal{A} \left(\overset{\circ}{R}_{\mu\nu} - \frac{1}{2} \overset{\circ}{R} g_{\mu\nu} \right) - (\mathcal{B} - \mathcal{C}') \phi_{,\mu} \phi_{,\nu} \\ & + \left(\frac{1}{2} \mathcal{B} - \mathcal{C}' \right) \phi_{,\rho} \phi_{,\sigma} g^{\rho\sigma} g_{\mu\nu} + \mathcal{C} \left(\overset{\circ}{\nabla}_\mu \overset{\circ}{\nabla}_\nu \phi - \overset{\circ}{\square} \phi g_{\mu\nu} \right) + \kappa^2 \mathcal{V} g_{\mu\nu} .\end{aligned}$$

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Simplified field equations

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- Trace-reversed symmetric tetrad field equations:

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- Effective gravitational constant:

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- ⇒ Corrections vanish for infinite mass $m_\phi \rightarrow \infty$.

- PPN parameters [Emtsova, MH '19]:

$$\gamma = 1 - \frac{C^2}{AB + 2C^2},$$

$$\beta = 1 - \frac{C\{6C^4(C+A') + ABC^2(7C+8A') + A^2[2B^2(C+A') + B'C^2 - 2BCC']\}}{4(AB+2C^2)^2(2AB+3C^2)},$$

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⇒ Identical to GR $\gamma = \beta = 1$ for $C = 0$.

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- Re-obtain well-known PPN parameters as consistency check.

Minimally coupled theories

- Numerous minimally coupled ($\mathcal{C} = 0$) theories:

- Teleparallel dark energy [Geng, Lee, Saridakis, Wu '11]:

$$\mathcal{A} = 1 + 2\kappa^2 \xi \phi^2, \quad \mathcal{B} = -\kappa^2.$$

- Interacting dark energy [Otalora '13]:

$$\mathcal{A} = 1 + 2\kappa^2 \xi F(\phi), \quad \mathcal{B} = -\kappa^2.$$

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⇒ Theories are indistinguishable from GR by PPN parameters.

Non-minimally coupled boundary term

- Action functional [Bahamonde, Wright '15]:

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- 1 Introduction
- 2 Parametrized post-Newtonian formalism
- 3 Teleparallel geometry and tetrad PPN formalism
- 4 PPN limits of teleparallel theories
 - Generic $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ teleparallel gravity
 - Scalar-torsion extension of scalar-curvature gravity
 - **Generic $L(T, X, Y, \phi)$ scalar-torsion gravity**
- 5 Conclusion

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- Taylor coefficients $l_0, l_T, l_X, l_Y, l_\phi, \dots$

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$$\begin{aligned} & -2\overset{\circ}{\nabla}_\rho (L_T \mathcal{S}_{\nu\mu}{}^\rho) - L_X \phi_{,\mu} \phi_{,\nu} + \overset{\circ}{\nabla}_\nu (L_Y \phi_{,\mu}) - \overset{\circ}{\nabla}_\sigma (L_Y \phi_{,\rho}) g^{\rho\sigma} g_{\mu\nu} \\ & - L_T \left(T^\rho{}_{\rho\sigma} T^\sigma{}_{\mu\nu} + 2T^\rho{}_{\rho\sigma} T_{(\mu\nu)}{}^\sigma - \frac{1}{2} T_{\mu\rho\sigma} T_\nu{}^{\rho\sigma} + T_{\mu\rho\sigma} T^{\rho\sigma}{}_\nu \right) \\ & + L_Y \left(T_{(\mu\nu)}{}^\rho \phi_{,\rho} + \frac{1}{2} T^\rho{}_{\mu\nu} \phi_{,\rho} + T^\rho{}_{\rho\mu} \phi_{,\nu} \right) - L g_{\mu\nu} = 2\kappa^2 \Theta_{\mu\nu} . \end{aligned}$$

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$$\begin{aligned} & -2\overset{\circ}{\nabla}_\rho (L_T \mathcal{S}_{\nu\mu}{}^\rho) - L_X \phi_{,\mu} \phi_{,\nu} + \overset{\circ}{\nabla}_\nu (L_Y \phi_{,\mu}) - \overset{\circ}{\nabla}_\sigma (L_Y \phi_{,\rho}) g^{\rho\sigma} g_{\mu\nu} \\ & - L_T \left(T^\rho{}_{\rho\sigma} T^\sigma{}_{\mu\nu} + 2T^\rho{}_{\rho\sigma} T_{(\mu\nu)}{}^\sigma - \frac{1}{2} T_{\mu\rho\sigma} T_\nu{}^{\rho\sigma} + T_{\mu\rho\sigma} T^{\rho\sigma}{}_\nu \right) \\ & + L_Y \left(T_{(\mu\nu)}{}^\rho \phi_{,\rho} + \frac{1}{2} T^\rho{}_{\mu\nu} \phi_{,\rho} + T^\rho{}_{\rho\mu} \phi_{,\nu} \right) - L g_{\mu\nu} = 2\kappa^2 \Theta_{\mu\nu}. \end{aligned}$$

- Scalar field equation:

$$g^{\mu\nu} \overset{\circ}{\nabla}_\mu (L_Y T^\rho{}_{\rho\nu} - L_X \phi_{,\nu}) - L_\phi = 0.$$

- PPN parameters [Flathmann, MH '19]:

$$\gamma - 1 = \frac{l_Y^2}{2l_T l_X - 2l_Y^2},$$

$$\beta - 1 = \frac{l_Y [l_T l_X l_Y^2 (16l_{T\phi} - 7l_Y) + 3l_Y^4 (l_Y - 2l_{T\phi}) - 8l_T^2 l_X^2 l_{T\phi} + 2l_T^2 l_Y (2l_X^2 + l_Y l_{X\phi} - 2l_X l_{Y\phi})]}{8(4l_T l_X - 3l_Y^2)(l_Y^2 - l_T l_X)^2},$$

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⇒ Identical to GR $\gamma = \beta = 1$ for $I_Y = 0$.

Analogue of scalar-curvature gravity

- Consider previous studied class of theories:

$$L(T, X, Y, \phi) = -\mathcal{A}(\phi)T + 2\mathcal{B}(\phi)X + 2\mathcal{C}(\phi)Y - 2\kappa^2\mathcal{V}(\phi).$$

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⇒ Consistency of both derivations.

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⇒ Theories are indistinguishable from GR $\gamma = \beta = 1$.

- 1 Introduction
- 2 Parametrized post-Newtonian formalism
- 3 Teleparallel geometry and tetrad PPN formalism
- 4 PPN limits of teleparallel theories
 - Generic $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ teleparallel gravity
 - Scalar-torsion extension of scalar-curvature gravity
 - Generic $L(T, X, Y, \phi)$ scalar-torsion gravity
- 5 Conclusion

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- Post-Newtonian limit of teleparallel gravity theories:
 - Obtained PPN parameters for different teleparallel theories.
 - All considered theories are fully conservative.
 - Large, widely used subclasses have same PPN limit as GR:
 - $f(T)$ theories with torsion scalar T fromTEGR.
 - Minimally coupled scalar-torsion theories with $C = 0$ or $l_\gamma = 0$.
 - $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ theories show no Nordvedt effect.

- Consider more general teleparallel gravity theories:
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 - Teleparallel extension to Horndeski gravity [Bahamonde, Dialektopoulos, Said '19].
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Outlook and work in progress

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- Extend formalism by including higher perturbation orders:
 - General covariant expansion instead of space-time split.
 - Allow also for fast-moving source masses.
 - Consider inspiral phase of black hole merger event.
 - Devise method for calculating gravitational waves.

- U. Ualikhanova and MH,
“Parameterized post-Newtonian limit of general teleparallel gravity theories”,
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- E. D. Emtsova and MH,
“Post-Newtonian limit of scalar-torsion theories of gravity as analogue to scalar-curvature theories”,
arXiv:1909.09355 [gr-qc].
- K. Flathmann and MH,
“Post-Newtonian limit of generalized scalar-torsion theories of gravity”,
arXiv:1910.01023 [gr-qc].

- Possibilities for study / research visits to Tartu.
 - Courses on gravity and differential geometry in English.
 - Active research group in teleparallel and related theories.
 - Large share of students - positive social environment.
- Exchange programs for master and PhD students.
- More information: <http://www.studyinestonia.ee>.