Energy and entropy in General Relativity

- or, the Canonical Frame in Q-Gravity

Tomi S. Koivisto

Laboratory of Theoretical Physics, Institute of Physics, University of Tartu
National Institute of Chemical Physics and Biophysics, Tallinn

tomik@astro.uio.no

09.10.2019, 4^{th} CANTATA Meeting, Tuzla

CANTATA
Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

Univerzitet u Tuzli
Bosna i Hercegovina

Displacement current of the metric

R. P. Feynman, *Feynman lectures on gravitation* (1962)

"Thus, gravity is that field which corresponds to a gauge invariance with respect to displacement transformations."

- Recall the original* EMT of the gravitational field:

$$t^{\mu}_{E\nu} = \frac{\partial L_E}{\partial g_{\alpha\beta,\mu}} g_{\alpha\beta,\nu} - \delta^{\mu}_{\nu} L_E \,.$$

- It is also the canonical† displacement current.

* A. Einstein, *Hamiltonsches Prinzip und allgemeine Relativitätstheorie* (1916)

† E. Noether, *Invariante Variationsprobleme* (1918)

Pseudo-tensors and complexes

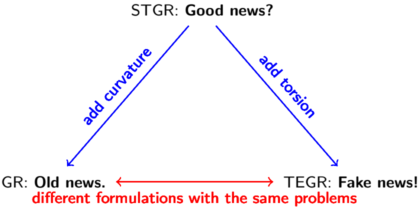
A. Einstein, *Hamiltonsches Prinzip und allgemeine Relativitätstheorie* (1916):

"Durch Ausführung der partiellen Integration" $\mathcal{R} \rightarrow$

$$L_E = g^{\mu\nu}\left(\left\{ {\alpha \atop \sigma\mu} \right\} \left\{ {\sigma \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \sigma\alpha} \right\} \left\{ {\sigma \atop \mu\nu} \right\} \right)$$

- The use of $\tau^{\mu}_{E\nu} = t^{\mu}_{E\nu} + T^{\mu}{}_{\nu}$ can be critised[‡].
- Many non-canonical alternatives have been proposed[§].

- In particular: metric teleparallelism does not remove, but replaces the problem.



[‡] Levi-Civita, Schrödinger, Weyl, Eddington, Pauli, etc

[§] Tolman, Landau-Lifshitz, Papapetrou-Gubta, Møller, Weinberg, etc

Affine connection

- Curvature: $R^\alpha{}_{\beta\mu\nu} = 2\partial_{[\mu}\Gamma^\alpha{}_{\nu]\beta} + 2\Gamma^\alpha{}_{[\mu|\lambda|}\Gamma^\lambda{}_{\nu]\beta}$
- Torsion: $T^\alpha{}_{\mu\nu} = 2\Gamma^\alpha{}_{[\mu\nu]}$
- Non-metricity: $Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu}$

$$\overbrace{\Gamma^\alpha{}_{\mu\nu}}^{\text{affine connection}} = \underbrace{\overbrace{\left\{ {\alpha \atop \mu\nu} \right\}}^{\text{Levi-Civita}}}_{-\frac{1}{2}g_{\mu\nu}{}^{,\alpha}+g^{\alpha\lambda}g_{\lambda(\mu,\nu)}} + \underbrace{\overbrace{K^\alpha{}_{\mu\nu}}^{\text{contortion}}}_{\frac{1}{2}T^\alpha{}_{\mu\nu}-T_{(\mu\nu)}{}^\alpha} + \underbrace{\overbrace{L^\alpha{}_{\mu\nu}}^{\text{disformation}}}_{\frac{1}{2}Q^\alpha{}_{\mu\nu}-Q_{(\mu\nu)}{}^\alpha}$$

Geometrical interpretation

- $\boldsymbol{R}_{[ab]} = \frac{1}{2}\text{ə}_a{}^\alpha\text{ə}_b{}^\beta R_{[\alpha\beta]\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$: how things turn around
- $\boldsymbol{T}^a = \frac{1}{2}\mathrm{e}^a{}_\alpha T^\alpha{}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$: how things are displaced
- $\boldsymbol{R}_{(ab)}$: how things change their size and shape

Affine curvature

- The curvature scalar can be decomposed accordingly:

$$\underbrace{R}_{\text{affine}} = \underbrace{\mathcal{R}}_{\text{metric}} + \underbrace{\mathbb{S}}_{\text{stuff}} + \underbrace{\mathcal{D}_\mu}_{\text{metric}} \Big(\underbrace{Q^\mu}_{Q^{\mu\alpha}{}_\alpha} - \underbrace{\tilde{Q}^\mu}_{Q^{\alpha\mu}{}_\alpha} + 2 \underbrace{T^\mu}_{T^{\alpha\mu}{}_\alpha} \Big)$$

- Therefore, for a flat connection with $R = 0$, we have

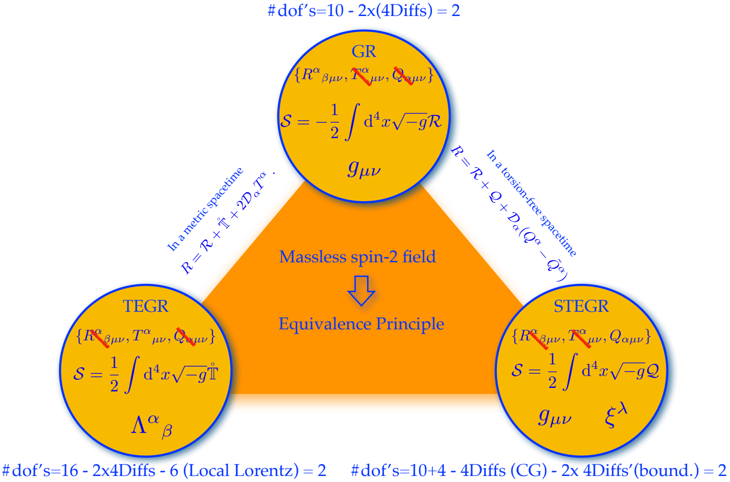
$$\underbrace{\frac{1}{2}M^2 \int \mathrm{d}^4 x \sqrt{-g}\mathcal{R}}_{\text{Einstein-Hilbert}} \underbrace{=}_{\text{up to a boundary term}} \underbrace{-\frac{1}{2}M^2 \int \mathrm{d}^4 x \sqrt{-g}\mathbb{S}}_{\text{Teleparallel equivalent}}$$

The post-Riemannian $\mathbb{S}$calar is the quadratic term

$$\mathbb{S} = \frac{1}{4} T_{\mu\nu\rho} T^{\mu\nu\rho} + \frac{1}{2} T_{\mu\nu\rho} T^{\mu\rho\nu} - T_\mu T^\mu + Q_{\mu\nu\rho} T^{\rho\mu\nu} - Q_\mu T^\mu + \tilde{Q}_\mu T^\mu + \frac{1}{4} Q_{\mu\nu\rho} Q^{\mu\nu\rho} - \frac{1}{2} Q_{\mu\nu\rho} Q^{\nu\mu\rho} - \frac{1}{4} Q_\mu Q^\mu + \frac{1}{2} Q_\mu \tilde{Q}^\mu$$

[Jose Beltrán Jiménez, Lavinia Heisenberg, Damianos Iosifidis, Alejandro Jiménez-Cano, TSK: arXiv:1909.09045]

The geometrical trinity [Jose Beltrán Jiménez, Lavinia Heisenberg, TSK: Universe 5 no.6, 153 (2019)]



(Re)interpretations

General Relativity

$$\ddot{x}^\alpha + \overbrace{\left\{ {\alpha \atop \mu\nu} \right\}}^{\text{geometry}} \dot{x}^\mu \dot{x}^\nu = 0\,.$$

Teleparallelism

$$\ddot{x}^\alpha + \overbrace{\Gamma^\alpha{}_{\mu\nu}}^{\text{Weitzenböck}} \dot{x}^\mu \dot{x}^\nu = \overbrace{K^\alpha{}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}^{\text{force} \quad F^\alpha}\,.$$

Symmetric teleparallelism

$$\ddot{x}^\alpha + \overbrace{\Gamma^\alpha{}_{\mu\nu}}^{\text{pure gauge}} \dot{x}^\mu \dot{x}^\nu = \overbrace{L^\alpha{}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}^{\text{inertia} \quad I^\alpha}\,.$$


"Q" IS HERE!
It's name is Quetzalcoatl...
just call it "Q"...
that's all you'll have time to say
before it tears you apart!
DAVID CARRADINE . RICHARD ROUNDTREE

Translation connection

- The generic affine connection $\Gamma^{\alpha}{}_{\mu\nu}$: 64 functions.
- The flat: $\Gamma^{\alpha}{}_{\mu\nu} = \Lambda^{\alpha}{}_{\beta}\partial_{\mu}(\Lambda^{-1})^{\beta}{}_{\nu}$: 16 functions.
- The flat and metric-compatible: $\Lambda^{\alpha\beta} = \Lambda^{[\alpha\beta]}$: 6 functions.
- The flat and symmetric: $\Lambda^{\alpha}{}_{\beta} = \partial_{\beta}\xi^{\alpha}$: 4 functions.

Jose Beltrán Jiménez, Lavinia Heisenberg, TSK: *Coincident General Relativity* (2017):

The affine connection of "**purified gravity**" is a pure translation; it has no geometry in the gauge-invariant sense. The connection vanishes in the "**coincident gauge**", $\mathring{\nabla}_{\mu} = \partial_{\mu}$.

Coincident General Relativity [J. Beltrán Jiménez, L. Heisenberg, TSK: PRD98, no.4 (2018)]

- There is a unique quadratic form which is doubly translation-invariant: $Q = -Q_\alpha{}^{\mu\nu} P^\alpha{}_{\mu\nu}$.

$P^\alpha{}_{\mu\nu} = -\frac{1}{4} Q_\alpha{}^{\mu\nu} + \frac{1}{2} Q_{(\mu}{}^{\alpha}{}_{\nu)} + \frac{1}{4}\left(Q^\alpha - \tilde{Q}^\alpha\right) g_{\mu\nu} - \frac{1}{4}\delta^\alpha_{(\mu} Q_{\nu)}$, where $Q_\alpha = Q_\alpha{}^\mu{}_\mu$, $\tilde{Q}^\alpha = Q_\mu{}^{\alpha\mu}$.

- At linear order, the connection decouples from the action.
- In the coincident gauge, Q reduces to L_E, $\mathring{Q} = -L_E$.

Recall, "durch Ausführung der partiellen Integration [...]" (1916)

$$\mathcal{R} \overset{\text{part. int.}}{\rightarrow} L_E = g^{\mu\nu}\left(\left\{ {\alpha \atop \sigma\mu} \right\} \left\{ {\sigma \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \sigma\alpha} \right\} \left\{ {\sigma \atop \mu\nu} \right\} \right) = -\mathring{Q} \overset{\text{gauge fix}}{\leftarrow} -Q$$

The coupling matters in a gauge theory.

The electromagnetic field:

$$\boldsymbol{F} = \partial_{[\mu} A_{\nu]} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \quad \overset{\text{M.C.}}{\rightarrow} \quad \boldsymbol{F} - \frac{1}{2} T^{\alpha}{}_{\mu\nu} A_\alpha \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \, .$$

Y. So, J. Nester, *On the source coupling and the teleparallel equivalent to GR* (2006):

"Can one decide which is the real 'physical' geometry? Invoking the minimal coupling principle may give a unique answer."

J. Itin, Yu. Obukhov, J. Boos, F. Hehl, *Premetric teleparallel theory of gavity and its local and linear constitutive law* (2018):

"Within the framework of TG as a translational gauge theory, the coupling to matter is achieved via the minimal coupling procedure, strictly in the sense of a bona fide gauge theory. Dispensing with the minimal coupling principle [...] is against the spirit of gauge theory."

Coupling matter: works in PG, doesn't in TG.

The hermitean Dirac action:

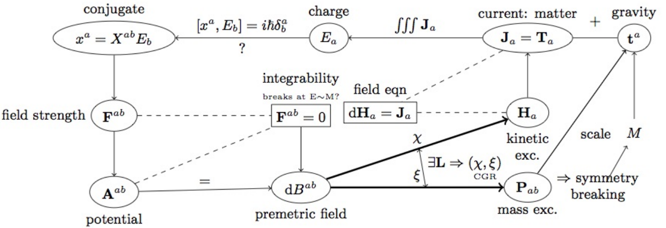
$$S_D = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[\left(i\bar{\psi}\gamma^\mu \mathrm{D}_\mu\psi\right) + \left(i\bar{\psi}\gamma^\mu \mathrm{D}_\mu\psi\right)^\dagger - 2m\bar{\psi}\psi\right] .$$

- Tangent space: $\mathrm{D}_\mu \mathbf{e}^a = 0 \quad \Rightarrow \quad \Lambda^a{}_{\mu b} = \mathrm{e}^a{}_\nu \left(\nabla_\mu \text{ə}_b{}^\nu\right)$.
- Spin connection: $\mathbf{\Gamma} = \frac{1}{4}\boldsymbol{A}_{ab}\gamma^a\gamma^b + \frac{1}{8}\boldsymbol{Z} = \frac{1}{4}\boldsymbol{A}_{ab}\gamma^{[a}\gamma^{b]} + \frac{1}{8}\boldsymbol{Q} + \frac{1}{8}\boldsymbol{Z}$.
- Hermiteanised: $\gamma \cdot \mathbf{\Gamma}^{\mathrm{H}} =$
  $-\frac{i}{4}\epsilon^{abcd}\mathrm{Re}(\boldsymbol{A}_{ab})\cdot \text{ə}_c\gamma_d\gamma^5 + \frac{i}{2}\mathrm{Im}(\boldsymbol{A}^{[ab]})\cdot \text{ə}_a\gamma_b - \frac{i}{8}\mathrm{Im}(\boldsymbol{Q}+\boldsymbol{Z})\cdot\gamma$.
- Decomposed: $i\Gamma^{\mathrm{H}}_\mu =$
  $\frac{1}{4}\mathrm{Re}\left(\tilde{\omega}_\mu + \tilde{T}_\mu\right)\gamma^5 - \frac{1}{2}\mathrm{Im}\left(\omega_\mu - T_\mu + \frac{1}{2}\tilde{Q}_\mu\right) + \frac{1}{8}\mathrm{Im}\left(3Q_\mu + Z_\mu\right)$.

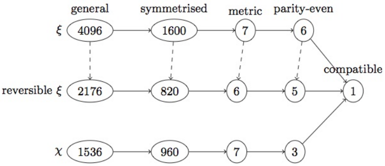
The hermitean Dirac equation:

$\left(i\gamma^\mu \mathrm{D}^H_\mu - m\right)\sqrt{-g}\psi = 0.$ The point is that though $\Gamma = \{\} + L \ \doteq\ 0$, spinors see only the $\{\}$.

Premetric Q-gravity [TSK, Manuel Hohmann, Luca Marzola: arXiv:1909.10415].



- Fundamental equations
  - conservation: $\mathrm{d}\boldsymbol{H}_a = \boldsymbol{J}_a$
  - integrability: $\boldsymbol{F}^{ab} = 0$
- Linking equations
  - kinetic: $\boldsymbol{A}^{ab} \xrightarrow{\chi} \boldsymbol{H}_a$
  - mass: $\boldsymbol{A}^{ab} \xrightarrow{\xi} \boldsymbol{P}_{ab}$



Analogy with massive electromagnetism

Extrapolation to Proca theory: impurities at the Planck scale M ?

$$L_{\text{em}} = \frac{1}{4} H^{\mu\nu} F_{\mu\nu} - \frac{1}{2} m^2 P^\mu A_\mu \,, \qquad L_{\text{gr}} = \frac{1}{4} H_{\alpha\beta}{}^{\mu\nu} F^{\alpha\beta}{}_{\mu\nu} - \frac{1}{2} M^2 P_{\alpha\beta}{}^{\mu} A^{\alpha\beta}{}_{\mu} \,.$$

Objects and laws	Basis	Electromagnetism	Gravity	W	Electromagnetism	Gravity
Source current	3-	$\mathbf{J} = \mathbf{T} + \boldsymbol{t}$	$\mathbf{J}_a = \mathbf{T}_a + \boldsymbol{t}_a$	1	$J^\mu = T^\mu + t^\mu{}_\nu$	$J^\mu{}_\nu = T^\mu{}_\nu + t^\mu{}_\nu$
Conservation law	4-	$\mathrm{d}\mathbf{J} = 0$	$\mathrm{d}\mathbf{J}_a = 0$	1	$\nabla_\mu J^\mu = 0$	$\nabla_\mu J^\mu{}_\nu = 0$
Kinetic excitation	2-	$\mathbf{H}$	$\mathbf{H}_a$, $\mathbf{H}_{ab}$	1	$H^{\mu\nu}$	$H^{\mu\nu}{}_\alpha$, $H^{\mu\nu}{}_{\alpha\beta}$
Mass excitation	3-	$\mathbf{P}$	$\mathbf{P}_{ab}$	1	P^α	$P^\alpha{}_{\mu\nu}$
Inhomog. field eqn.	3-	$\mathrm{d}\mathbf{H} = \mathbf{J}$	$\mathrm{d}\mathbf{H}_a = \mathbf{J}_a$	1	$\nabla_\mu H^{\mu\nu} = J^\nu$	$\nabla_\mu H^{\mu\nu}{}_\alpha = J^\mu{}_\alpha$
Kinetic potential	1+	$\mathbf{A}$	$\mathbf{A}^{ab}$	0	A_μ	$A^{\alpha\beta}{}_\nu$
Mass potential	0+	B	B^{ab}	0	B	$B^{\alpha\beta}$
Field strength	2+	$\mathbf{F} = \mathrm{d}\mathbf{A}$	$\mathbf{F}^{ab} = \mathrm{d}\mathbf{A}^{ab}$	0	$F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$	$F^{\alpha\beta}{}_{\mu\nu} = 2\nabla_{[\mu} A^{\alpha\beta}{}_{\nu]}$
Homog. field eqn	3+	$\mathrm{d}\mathbf{F} = 0$	$\mathrm{d}\mathbf{F}^{ab} = 0$	0	$\nabla_{[\alpha} F_{\mu\nu]} = 0$	$\nabla_{[\sigma} F^{\alpha\beta}{}_{\mu\nu]} = 0$
Lorentz force	4-	$f_a = ə_a \cdot \mathbf{F} \wedge \mathbf{J}$	$f^c{}_a = ə_a \cdot \mathbf{F}^{cd} \wedge \mathbf{J}_d$	1	$f_\mu = F_{\mu\nu} J^\nu$	$f^\alpha{}_\mu = F^{\alpha\beta}{}_{\mu\nu} J^\nu{}_\beta$
Effective force	4-	$\mathfrak{f} = -\mathrm{d}\boldsymbol{t}$	$\mathfrak{f}_a = -\mathrm{d}\boldsymbol{t}_a$	1	$\mathfrak{f} = -\nabla_\mu t^\mu$	$\mathfrak{f}_\nu = -\nabla_\mu t^\mu{}_\nu$
Energy-momentum	3-	${}^{\text{em}}\mathbf{T}_a$	$\boldsymbol{t}_a$	1	${}^{\text{em}}T^\mu{}_\nu$	$t^\mu{}_\nu$
Kinetic Lagrangian	4-	${}^{\text{kin}}\boldsymbol{\Lambda} = -\frac{1}{2}\mathbf{F} \wedge \mathbf{H}$	${}^{\text{kin}}\boldsymbol{\Lambda} = -\frac{1}{2}\mathbf{F}^{ab} \wedge \mathbf{H}_{ab}$	1	${}^{\text{kin}}L = \frac{1}{4} H^{\mu\nu} F_{\mu\nu}$	${}^{\text{kin}}L = \frac{1}{4} H^{\mu\nu}{}_{\alpha\beta} F^{\alpha\beta}{}_{\mu\nu}$
Mass Lagrangian	4-	${}^{\text{pot}}\boldsymbol{\Lambda} = -\frac{1}{2}\mathbf{A} \wedge \mathbf{P}$	${}^{\text{pot}}\boldsymbol{\Lambda} = -\frac{1}{2}\mathbf{A}^{ab} \wedge \mathbf{P}_{ab}$	1	${}^{\text{pot}}L = -\frac{1}{2} P^\mu A_\mu$	${}^{\text{pot}}L = -\frac{1}{2} P^\alpha{}_{\mu\nu} A^{\mu\nu}{}_\alpha$
Kinetic constitutive rel.	0-	χ^{abcd}	$\chi^{ab}{}_{cdf}{}^e$, $\chi^{abh}{}_{cdfi}{}^e$	1	$\chi^{\alpha\beta\mu\nu}$	$\chi^{\mu\nu}{}_{\alpha\beta\gamma}{}^\delta$, $\chi^{\mu\nu\sigma}{}_{\alpha\beta\gamma\rho}{}^\delta$
Mass constitutive rel.	0-	ξ^{ab}	$\xi^a{}_{cd}{}^b{}_{ef}$	1	$\xi^{\alpha\beta}$	$\xi^\alpha{}_{\mu\nu}{}^\beta{}_{\rho\sigma}$

> "The Standard Model is the result of the efforts to extend the ideas and methods of the electromagnetic interactions to all other forces in physics." -J. Iliopoulos, (2012).

A new view at the Einstein equation

$$\mathrm{D}\boldsymbol{H}_a = \boldsymbol{T}_a + \boldsymbol{t}_a$$

$\mathcal{G}^\mu{}_\nu$ =	$\tau^\mu{}_\nu$ −	$t^\mu{}_\nu$ =	$T^\mu{}_\nu$
Einstein tensor	Gravitational EMT	Inertial EMT	Matter EMT
$\mathcal{R}^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu\mathcal{R}$	$\frac{1}{\sqrt{-g}}\nabla_\alpha\left(\sqrt{-g}H^{\alpha\mu}{}_\nu\right)$	$\frac{1}{2}Q\delta^\mu_\nu - P^\mu{}_{\alpha\beta}Q_\nu{}^{\alpha\beta}$	$-g_{\alpha\nu}\frac{2\delta(\sqrt{-g}L_M)}{\sqrt{-g}\delta g^{\mu\alpha}}$
co-covariant	$\rightarrow$ "Einstein EM complex"	$\rightarrow$ $\mathring{t}^\mu{}_\nu = t^\mu_{E\,\nu}$	co-covariant

Jose Beltrán Jiménez, Lavinia Heisenberg, TSK: *The Canonical Frame of Purified Gravity* (IJMPD 2019):

The original $H^{\mu\nu}{}_\alpha$ is vindicated in the now covariant formulation.
The new conjecture: **the canonical frame** is $t^\mu{}_\nu = 0$.

- The field equations are both invariant and "pseudo-invariant"
- Their decomposition is not "pseudo-invariant"

The energy and the entropy

Canonical frame: $\boldsymbol{t}_a = 0$

$$\mathrm{D}\boldsymbol{H}_a = \boldsymbol{T}_a + \not{\boldsymbol{t}}_a$$

The physical principle that the metric should not contribute to the energy-momentum, determines uniquely the energetics and thermodynamics in a gravitating system.

- **Energy-momentum** from Noether's theorem:
  $\mathring{P}_\mu = \int \mathrm{d}^2 S \sqrt{-g} \mathring{H}^{i0}{}_\mu n_i$.
- **Entropy** from Euclidean QG:
  $-\log Z \approx \frac{1}{2} \int \mathrm{d}^3 V \sqrt{-g} \left(Q^\alpha - \tilde{Q}^\alpha \right) n_\alpha + \int \mathrm{d}^4 x \sqrt{-g} \left(L_M - \frac{1}{2} T \right)$.
- **Consistency** from thermodynamics:
  $S = \beta E_0 + \log Z$.

Results for black holes (in natural units)

- The computation of Schwarzschild black hole entropy
  - in GR: $S \approx \frac{1}{16\pi}\int \underbrace{\mathrm{d}^4x\sqrt{-g}\mathcal{R}}_{=0} + \underbrace{\int \mathrm{d}^3x\mathcal{K}}_{=\infty} - \underbrace{\int \mathrm{d}^3x\mathcal{K}_0}_{=\infty} = 4\pi M^2$.
  - in Q-gravity: $S \approx -\frac{1}{16\pi}\int \mathrm{d}^3x\tilde{Q}^\mu n_\mu = 4\pi M^2$.
- For the energy we obtain consistently $E_\mu = M\delta^0_\mu$
  - The computations are covariant, the results unique
  - ~~counter terms~~, ~~boundary terms~~, ~~higher derivatives~~, ~~ambiguity~~
- Example 2: Reissner-Nordström black hole
  - The Euclidean action gives $-\log Z \approx \pi r_+^2 + \beta q^2/(2r_+)$,
  - and we obtain for the canonical energy $E_0 = M - q^2/(2r_+)$.

Cosmological results (in progress...)

- Example 3: de Sitter space
  - The Euclidean action gives $-\log Z \approx \pi r_+^2$,
  - and we obtain the canonical energy-momentum $E_\mu = 0$.
- Example 4: general FLRW
  - I haven't found the solution in the canonical frame yet and
  - can only speculate about the Tryon hypothesis.
- Example 5: Gravitational waves
  - No ρ nor p but momentum: $P_\mu = \frac{1}{2}\delta^j_\mu \int \mathrm{d}^2 S \sqrt{-g}\dot{h}^i{}_j n_i$
  - Cooperstock hypothesis survives Feynman's bead argument.

Hypercomplex and holographic magic

- We couple to the vanishing $\Gamma = \{\} + L \doteq 0$, yet follow the geodesics of $\{\}$.

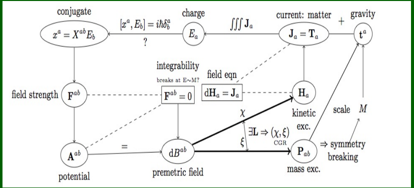
- The vacuum with vanishing energy density may have a finite energy charge.

- The integral of a vanishing action may describe a finite entropy.

Conclusion

Q-gravity is

- the canonical
  - Geometrically: $\mathring{Q} = -L_E$
  - Materially: MCP ok
- translation gauge theory
  - TGR no more so than GR
  - In STGR Γ is a translation
- which solves the *E*-problem
  - Covariant energy
  - Improved path integral



$$L = -\frac{1}{2}\boldsymbol{F}^{ab} \wedge \boldsymbol{H}_{ab} - \frac{1}{2}M^2\boldsymbol{A}^{ab} \wedge \boldsymbol{P}_{ab}$$

- and offers a new approach to resolve the conjugate *t*-problem.
  - Complete analogy with Proca electromagnetism
  - Generalisation of spacetime @ℓ_{Planck} : $\boldsymbol{Q}^{ab} \to \boldsymbol{A}^{ab} \neq -\mathrm{D}\eta^{ab}$!